A Textbook on

Elements of Civil Engineering and Engineering Mechanics

(As per VTU 06 CIV 13/06 CIV 23 Syllabus)

(Second Edition)

S.S. Bhavikatti
Principal, Vijayanagar Engineering College, Bellary
Formerly, Professor and Dean
SDM College of Engg. & Technology, Dharwad and
National Institute of Technology, Surathkal

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Preface to the Second Edition

Thanks to the enthusiastic support from the readers of first edition and their suggestions to correct some of the print mistakes. VTU brought out two model question papers and conducted one examination according to 06 CIV syllabus in the month of January 2007. To train the students to write answers properly author has included those two model question papers and answers to them. VTU question paper of January 2007 may be taken as another model question paper which is presented here as model question paper III and for that answers to numerical problems are presented. A student can try to solve it and check his/her ability to get correct answers.

S.S. Bhavikatti

Preface to the First Edition

Visveswaraya Technological University Belgaum has a compulsory subject for all engineering students in the subject Elements of Civil Engineering and Engineering Mechanics. This course has been revised for the academic year 2006 onward. It includes few topics from general civil engineering, roads, bridges and dams. Major portion of this book is engineering mechanics. A student cannot afford to procure the books pertaining to all these courses in civil engineering and make use of them. Hence there is need for special book for this course, and hence the author is bringing out this book.

First three chapters are on general civil engineering and author has gone to depth just requires by the VTU syllabus. Next eight chapters are on engineering mechanics. In these chapters neat free body diagrams are drawn and problems are solved systematically to make the procedure clear. Throughout the text SI units and standard notations as recommended by Indian Standard Codes are used.

Suggestions for improvements are most welcome from students and teachers.

S.S. Bhavikatti

Syllabus

ELEMENTS OF CIVIL ENGINEERING & ENGINEERING MECHANICS

Sub Code	:	06 CIV 13/06 CIV 23	**IA Marks**	:	25
Hrs/Week	:	04	**Exam Hours**	:	03
Total Hrs.	:	52	**Exam Marks**	:	100

PART – A

Unit-I

1. Introduction to Civil Engineering, Scope of different Field of Civil Engineering - Surveying, Building Materials, Construction Technology. Geotechnical Engineering, Structural Engineering, Hydraulics, Water Resources and Irrigation Engineering, Transportation Engineering, Environmental Engineering.

   Infrastructure: Types of Infrastructure, Role of Civil Engineer in the Infrastructural Development, Effect of the Infrastructural Facilities on Socio-economic Development of a Country.

   04 Hours

2. Roads: Type of roads, Components and their functions.

   02 Hours

3. Bridges and Dams: Different types with simple sketches.

   01 Hours

Unit-II

4. Introduction to Engineering mechanics: Basic idealisations - Particle, Continuum, Rigid body and Point force; Newton's law of motion, Definition of force, Introduction to SI units, Elements of a Force, Classification of force and Force systems; Principale of physical independence of forces, Principle of superposition of forces, Principle of transmissibility of forces; Moment of a force, couple, moment of a couple, characteristics of couple, Equivalent force - couple system; Resolution of forces, composition of forces: Numerical problems on moment of forces and couples, on equivalent force - couple systems.

   07 Hours

Unit-III

5. Composition of forces - Definition of Resultant Composition of coplanar-concurrent force system, Principle of resolved parts; Numerical problems on composition of coplanar concurrent force systems.

03 Hours

6. Composition of coplanar-non-concurrent force system, Varignon's principle of moments; Numerical problems on composition of coplanar non-concurrent force systems.

05 Hours

Unit-IV

7. Centroid of plane figures; Locating the centroid of triangle, semicircle, quadrant of a circle and sector of a circle using method of integration, Centroid of simple built up sections; Numerical problems.

06 Hours

PART-B

Unit-V

8. Equilibrium of forces - Definition of Equilibrant; Conditions of static equilibrium for different force system. Lami's theorem; Numerical problems on equilibrium of coplanar—concurrent force system.

06 Hours

Unit-VI

9. Types of supports, statically determinate beams, Numerical problems on equilibrium of coplanar—non-concurrent force system and support reactions for statically determinate beams.

06 Hours

Unit-VII

10. Friction—Types of friction, Laws of static friction, Limiting friction. Angle of friction, angle of repose; Impending motion on horizontal and inclined planes; Wedge friction; Ladder friction; Numerical problems.

06 Hours

Unit-VIII

11. Moment of inertia of an area, polar moment of inertia, Radius of gyration, Perpendicular axis theorem; Moment of Inertia of rectangular, circular and triangular areas from method of integration; Moment of inertia of composite areas; Numerical problems.

Contents

CHAPTER 1

Introduction to Civil Engineering

Civil engineering is the oldest branch of engineering which is growing right from the stone age civilization. American Society of Civil Engineering *defines Civil Engineering as the profession in which a knowledge of the mathematical and physical sciences gained by study, experience and practice is applied with judgement to develop ways to utilize economically the materials and forces of nature for the progressive well-being of man.*

In this chapter, scopes of different fields of civil engineering are discussed and the importance of developing infrastructure in the country is presented.

1.1 SCOPE OF DIFFERENT FIELDS OF CIVIL ENGINEERING

Civil Engineering may be divided into the following fields:

(i) Surveying
(ii) Building Materials
(iii) Construction Technology
(iv) Structural Engineering
(v) Geotechnical Engineering
(vi) Hydraulics
(vii) Water Resources and Irrigation Engineering
(viii) Transportation Engineering
(ix) Environmental Engineering and
(x) Town planning

Scope of each one of these is discussed below.

(i) Surveying

Surveying is the science of map makng. To start any development activity in an area the relative positions of various objects in the horizontal and vertical directions are required. This is approved by surveying the area. Earlier, the conventional instruments like chain, tape and levelling instruments were used. In this electronic era, modern equipments like distance meters and total stations are used to get more accurate results easily. The modern technologies like photogrammetry and remote sensing have made surveying easier.

(ii) Building Materials

Shelter is the basic need of civilization. To get good shelter continuous efforts are going on right from the beginning of civilization. Stones, bricks, timber, lime, cement, sand, jellies and tiles are the traditional building materials. Use of steel, aluminium, glass, glazed tiles, plaster of paris, paints and varnishes have improved the quality of buildings. The appropriate mixture of binding materials like lime and cement with sand is known as mortar. The mixture of cement, sand and jelly (crushed stones) with water is known as concrete. The use of concrete with steel bars placed in appropriate position has helped in building strong and durable tall structures. The composite material of concrete and steel is called reinforced cement concrete which is popularly known as R. C C. A civil engineer must know the properties of all the building materials so that they can be used appropriately. Improved versions of many building materials appear in the market. A good civil engineer will make use of them at the earliest.

(iii) Construction Technology

Construction is the major activity of civil engineering which is continuously improving. As land cost is going up there is demand for tall structures in urban areas while in rural areas need is for low cost constructions. One has to develop technology using locally available materials. In India, contribution of Central Building Research Institute (CBRI) - Roorkee and Gaziabad, several educational institutions throughout the country and Nirmithi Kendras in the technology development are noteworthy.

(iv) Structural Engineering

Load acting on a structure is ultimately transferred to ground. In doing so, various components of the structure are subjected to internal stresses. For example, in a building, load acting on a slab is transferred by slab to ground through beams, columns and footings. Assessing the internal stresses in the components of a structure is known as Structural Analysis and finding the suitable size of the structural component is known as design of structure. The structure to be analysed and designed may be of masonry, R. C C. or steel. Upto midsixties considerable improvements were seen in classical analysis. With the advent of computers numerical methods emerged and analysis and design packages are becoming popular. Matrix Method of analysis and Finite Elements Analysis have helped in the analysis of complex structures. A civil engineer has not only to give a safe structure but he has to give economical sections. To get economical section mathematical optimization techniques are used. Frequent earthquakes in the recent years have brought, importance of analysis of the structure for earthquake forces. Designing earthquake resistant structures is attracting lot of researches. All these aspects fall under structural engineering field.

(v) Geotechnical Engineering

Soil property changes from place to place. Even in the same place it may not be uniform at various depths. The soil property may vary from season to season due to variation in moisture content. The load from the structure is to be safely transferred to soil. For this, safe bearing capacity of the soil is to be properly assessed. This branch of study in Civil Engineering is called as Geotechnical Engineering.

Apart from finding safe bearing capacity for foundation of buildings, geotechnical engineering involves various studies required for the design of pavements, tunnels, earthen dams, canals and earth retaining structures. It involves study of ground improvement techniques also.

(vi) Hydraulics

Water is an important need for all living beings. Study of mechanics of water and its flow characteristics is another important field in Civil Engineering and it is known as hydraulics.

(vii) Water Resources and Irrigation Engineering

Water is to be supplied to agriculture field and for drinking purposes. Hence suitable water resources are to be identified and water is to be stored. Identifying, planning and building water retaining structures like tanks and dams and carrying stored water to fields is known as water resources and irrigation engineering.

(viii) Transportation Engineering

Transportation facility is another important need. Providing good and economical roads is an important duty of civil engineers. It involves design of base courses, suitable surface finishes, cross drainage works, road intersections, culverts, bridges, tunnels etc. Railway is another important long-way transport facility. Design, construction and maintenance of railway lines, signal system are part of transportation engineering. There is need for airports and harbours. For proper planning of these transportation facility, traffic survey is to be carried out. Carrying out traffic survey, design, construction and maintenance of roads, bridges, railway, harbour and airports is known as transportation engineering.

(ix) Environmental Engineering

Proper distribution of water to rural areas, towns and cities and disposal of waste water and solid waste are another field of civil engineering. Industrialisation and increase in vehicular traffic are creating air pollution problems. Environmental engineering while tackling all these problems provides healthy environment to public.

(x) Architecture and Town Planning

Aesthetically good structures are required. Towns and cities are to be planned properly. This field of engineering has grown considerably and has become a course separate from Civil Engineering.

1.2 TYPES OF INFRASTRUCTURE

Infrastructure facilities involve various civil engineering amenities, electricity, telephone, internet facility, educational and healthcare facilities. Civil engineering amenities in the infrastructure developments are listed below:

(i) A good town planning and developing sites
(ii) Providing suitable roads and network of roads
(iii) Railway connection to important places

(iv) Airports of national and international standards
(v) Assured water supply to towns, cities and rural areas
(vi) A good drainage and waste disposal system
(vii) Pollution free environment.

1.3 EFFECT OF INFRASTRUCTURE FACILITIES

Connecting producing centre to marketing places minimises exploitation from middlemen. Both producer and consumers are benefitted. Imports and exports become easy as a result of which whole world becomes a village. The infrastructure development generates scope for lots of industries. Manpower is utilized for the benefit of mankind. Antisocial activities come under control. Improved education and healthcare give rise to skilled and healthy work force. Quality of life of the people is improved. In case of natural calamities assistance can be extended easily and misery of affected people is reduced. Infrastructure facility improves defence system and peace exists in the country. Improved economical power of the country brings a respectable status in the world.

The world has realized that a government should not involve itself in production and distribution but should develop infrastructure to create an atmosphere for economical development.

1.4 ROLE OF CIVIL ENGINEERS IN THE INFRASTRUCTURE DEVELOPMENT

A civil engineer has to conceive, plan, estimate, get approval, create and maintain all civil engineering infrastructure activities. He has to carry out research and training programmes to improve the technology. Civil engineer has a very important role in the development of the following infrastructures:

(i) Town and city planning
(ii) Build suitable structures for the rural and urban areas for various utilities.
(iii) Build tanks, dams to exploit water resources.
(iv) Purify the water and supply water to needy areas like houses, schools, offices, and agriculture field.
(v) Provide good drainage system and purification plants.
(vi) Provide and maintain communication systems like roads, railways, harbours and airports.
(vii) Monitor land, water and air pollution and take measures to control them.

Questions

1. Briefly give the scope of different fields in Civil Engineering.
2. List various civil engineering amenities covered under infrastructure developments.
3. Discuss briefly impact of Civil Engineering infrastructure developments on the economy and environment.
4. Briefly explain the role of Civil Engineers in the infrastructure development.

CHAPTER 2

Roads

The path overwhich vehicles and other traffic may lawfully pass is called road. It includes pathway, other related structures like culverts, bridges and land aquired for future widening. The entire area required and reserved for road along its alignment is called *Right of Way*.

Development of civilisation is closely associated with the developments of roads. The first hard surface was constructed in 3500 BC in Mesopotamia. At the same time even in Mohenjodaro and Harappa well built roads were seen. In 600 BC a metallic road of 6 to 7.5 m wide existed in Rajgir near Patna. In about 300 BC, Kautilya got constructed National Highway connecting North West Frontier Province (now in Pakistan) and Patna. Chandra Gupta Maurya and Ashoka contributed considerably for development of road network. Mohammed Tughlaq constructed a road connecting Delhi and Doulatabad. Shershah constructed a Highway from Lahore to Sonargaon (Bengal). In the British rule Lord William Bentinck, Lord Dalhousie, Lord Minto and Lord Ripon contributed a lot for the road development in India.

In 1934 a semi-official technical body known as Indian Road Congress (IRC) was formed to provide a forum for regular pooling of experience and ideas on all matters affecting the planning, construction and maintenance of roads in India. First 20 years road development plan was prepared for whole country in the 1943 conference held at Nagpur. It is known as Nagpur Road Plan. It classified various roads required for the country and aimed at achieving a target of 16 km per 100 square kilometre of the country, by 1963. The second twenty years road development plan was finalised in the IRC meeting held in Bombay in 1959. It aimed at providing 32 km length of road per 100 square kilometre area. This is known as Bombay Road Plan and it covered 20 years from 1961 to 1981. The third 20 years development plan for the period 1981-2001 was finalised in Golden Jubilee Conference of IRC in 1985 held at Lucknow, It aimed at achieving a road density of 82 km per 100 sq. m area.

2.1 TYPES OF ROADS

Various criteria may be used for classifying roads. Depending upon the usage of roads during rainy season they may be classified as

(i) all weather roads and

(ii) fair weather roads.

All weather roads are not flooded during rainy seasons except to a small extent at river crossing for a small length. In fair weather roads overflowing of streams across the road is permitted during monsoon season.

Based on the type of pavement surfaces provided the roads may be classified as

(i) surfaced roads and

(ii) unsurfaced roads.

Surfaced roads are provided with a bituminous or concrete surface while unsurfaced roads may be mud roads or water bound macadam layer roads.

The **Nagpur Road Plan** classified the roads in India into the following five categories:

(i) National Highways (NH)

(ii) State Highways (SH)

(iii) Major District Roads (MDR)

(iv) Minor or other District Roads (ODR) and

(v) Village Roads (VR)

(i) National Highways (NH)

These are the roads connecting important cities, towns, ports etc. of different states. They may even connect the neighbouring countries also. The National Highways have two-lane traffic at least 8 m wide with at least 2 m wide shoulders on each side. The construction and maintenance of these roads is taken care by the Central Government agencies like Central PWD or Military Engineering Service (MES). The National Highways are assigned the respective numbers. The highway connecting Delhi-Ambala-Amritsar is denoted as NH-1. The National Highway connecting Poona-Bangalore-Chennai is called as NH-4. The west-coast highway connecting Bombay to Kanyakumari is known as NH-17.

(ii) State Highways (SH)

These are important roads of a particular state connecting important cities and district headquarters. They connect important cities to national highways. They are maintained by State Public Works Departments and Central Government gives grants for the construction and development of these roads. These highways also have 8 m carriage way and 2 m wide shoulders on each side. The design speed and design specifications of State Highways are same as those for National Highways.

(iii) Major District Roads (MDR)

These are the roads within a district connecting market and production areas to State or National Highways or railway stations. The MDR has lower speed and geometric design specifications than for NH or SH.

(iv) Minor or Other District Roads (ODR)

These roads connect rural areas of production to market centres, taluk centres or other main roads. These roads have lower design specifications than MDR. These roads are looked after by district authorities with the help of State Government Departments.

(v) Village Roads (VR)

The roads connecting villages or group of villages with each other or the roads of higher category. The local district boards are responsible for the construction and maintenance of these roads. These roads are usually unmetalled.

After the **third road development plan (Lucknow Road Plan)** the roads in the country are classified into three classes, viz.

(i) **Primary System**

(ii) **Secondary System** and

(iii) **Tertiary system** or **Rural Roads**

Primary system consists of two categories

(i) Expressways and

(ii) National Highways (NH)

Expressways are superior to National Highways and are provided wherever volume of traffic is very high. They have superior facilities and design speed. No cross-traffic is permitted on Expressways. They are provided with central separator for the traffic in opposite directions. They are fenced so that animals do not enter. Controlled access is provided to other roads and cities and towns. Only fast moving vehicles are permitted. Expressways may be owned by State or Central Government. Golden Quadrilateral connecting Delhi, Bombay, Chennai, Calcutta and Delhi is owned by Central Governments. This line passes through Belgaum, Dharwad-Hubli, Davangere, Chitradurga, Tunkur and Bangalore in Karnataka, Bangalore–Mysore Infrastructure Corridor (BMIC) expressway, which is under construction, is owned by the State Government. The construction works have been under taken on the basis of Build–Operate–Transfer (BOT) by private parties on contract assigned by the respective governments.

The secondary system consists of two categories of roads, namely, State Highways and Major District Roads.

The third category of roads consist of Other District Roads and Village Roads.

Urban Roads form a separate class of roads, which are taken care by municipalities/ municipal corporations. These roads may consist of the following:

(i) Expressways

(ii) Arterial roads

(iii) Sub-arterial Roads

(iv) Collector streets and

(v) Local streets.

Local streets are abutt to private properties like shops and houses. They are connected to collector streets. Arterial and sub-arterial roads are streets primarily to through traffic.

2.2 COMPONENTS OF ROADS

All roads consist of the following components:

(i) Pavement or carriageways

(ii) Shoulders

(i) Pavement or Carriageways

This is the width of road which is designed to handle volume of expected traffic. As per Indian Road Congress specification, the maximum width of vehicle is 2.44 m. A side margin of 0.68 m is required for safe drive of the vehicle. Hence for a single lane road carriageway width works out to be 3.8 m. For road pavements having two or more lanes, the width of 3.5 m per lane is considered sufficient. Number of lanes required for a road is decided by volume of traffic to be handled and also financial considerations. The cross-section of carriageway consists of the following components [Ref. Fig. 2.1]:

(a) Subsoil
(b) Subgrade
(c) Base and
(d) Surfacing.

Subsoil is the natural or prepared soil which will take the load of the road. It is prepared by properly compacting the natural soil.

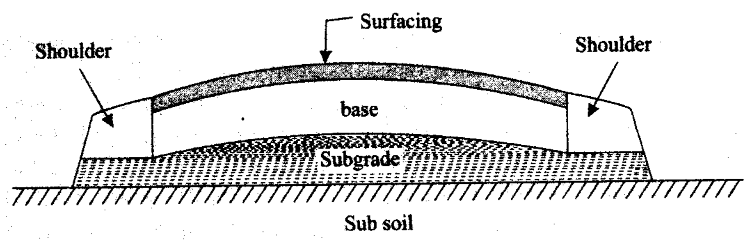


Fig. 2.1 Cross-section of carriageway

Subgrade gives support to the road structure. It should remain stable and dry throughout. Considerable attention is to be given to laying proper subgrade to get stable road surface. The subgrade soil consists mainly of disintegrated rocks like gravel, sand, silt and clay. The desirable properties of subgrade soil are

- stability
- permanency of strength
- incompressibility
- minimum change in volume
- ease of compaction and
- good drainage.

The **base** may consist of two layers, top layer being called as base and bottom one as sub-base. Base course and sub-base course distribute the load through a finite thickness. Sub-base is made with stabilised soil or selected granular soil, bricks or boulders. However, it is better if graded aggregates with soil are used instead of boulders. Base course is provided with broken stone aggregates.

Surfacing is the topmost layer of carriageway which takes load from traffic directly. It has to provide a smooth nonslippery and stable surface for the vehicles. It should be impervious and should protect base and subgrade from rainwater. It may be provided with bituminous material or with cement mixed with baby jelly.

(ii) Shoulders

The width of carriageway is extended on both sides of carriageway by a minimum of 2 - 5 m. It acts as service lane for the broken down vehicle and in case of blocking of carriageway it serves as emergency lane. The requirements of shoulders are

(a) its colour should be different from that of pavement surface so that they are distinct in vision.

(b) they should have sufficient load bearing capacity so as to support loaded trucks in wet weather also.

(c) surface of shoulder should be rough compared to pavement so that drivers are discouraged to use it as regular lane.

Other Components of Roads

Some of the roads will be having the following components also:

(a) traffic separators

(b) kerbs

(c) footpaths

(d) parking lanes

(e) cycle tracks

(f) gaurd rails and

(g) fencing

Traffic separators are provided to separate the traffic moving in apposite directions. It avoids head on collision between vehicles moving in opposite directions. Traffic separator may be in the form of pavement marking or parkway strips whose width vary from 3 to 5 m. If width is to be reduced due to unavoidable situations 1 in 15 to 1 in 20 transitions are provided.

Kerbs are provided to show the boundary between carriageway and shoulder or footpaths. They provide lateral stability to the base course. There are three classes of kerbs which are based on the height of the kerb and its function.

Class I kerbs are known as low kerbs or mountable kerbs. Their height with respect to pavement edge varies from 70 to 80 mm. These kerbs permit vehicles to mount on in case of emergency.

Class II kerbs are known as urban parking kerbs or low speed barriers. Their height above pavement edge varies from 150 mm to 200 mm. They are provided with 25 mm batter to prevent scrapping of tyres of vehicle. These kerbs discourage encroachment of slow speed vehicles but at the same time, in case of acute emergency, permit parking of vehicles with some difficulty.

Class III kerbs are known as high speed barriers also. Their height varies from 230 mm to 450 mm. They prevent vehicles leaving carriageways. They are usually provided in hill roads and bridges.

Footpaths are provided for pedestrian to separate them from vehicular traffics. They are usually required in city roads. The width of footpath is kept 1.3 m or more, depending upon volume of pedestrian traffic. To encourage pedestrian to use footpath, surface should be smooth and comfortable.

Sometimes **parking lanes** are provided in cities to streamline vehicle parking.

In urban areas, if cycles are also popular, separate **cycle tracks** are provided. Usually a minimum width of 2 m cycle tracks are provided.

When the height of fill exceeds 3 m, the **gaurd rails** are provided on the edge of shoulders to avoid accidental fall of vehicles down the fill.

In express Highways **fencing** is provided to prevent animals or other traffic entering roads haphazardly.

Typical cross-sections of various roads with their components are shown in Figs. 2.2 to 2.6.

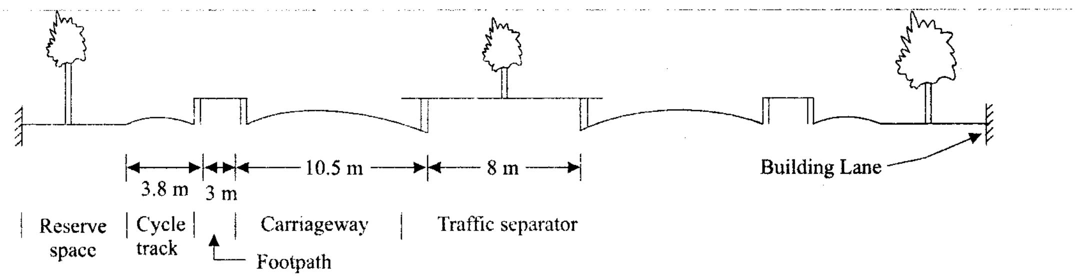


Fig. 2.2 Cross-section of divided highway in urban area

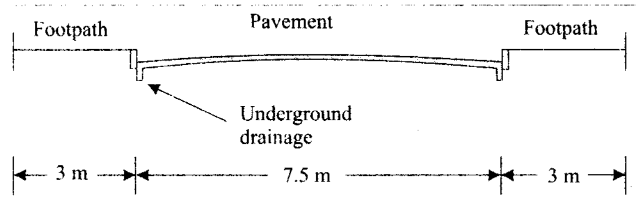


Fig. 2.3 Cross-section of city road in built-up area

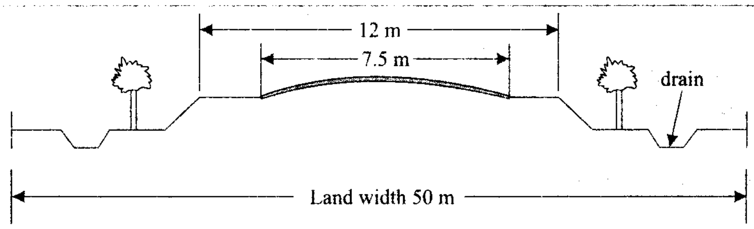


Fig. 2.4 Cross-section of two-lane NH or SH in rural area

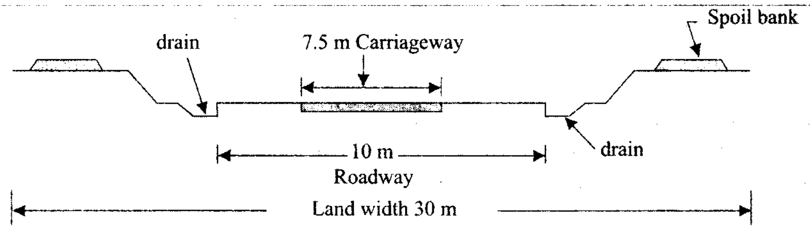


Fig. 2.5 Cross-section of MDR in cutting

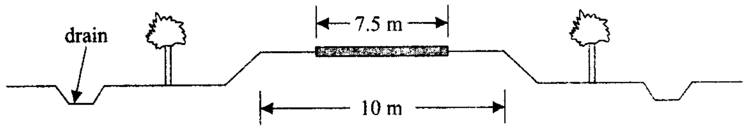


Fig. 2.6 Cross-section of MDR in embankment

Questions

1. Briefly outline the history of road construction in India.
2. Distinguish between
   (i) all weather roads and fair weather roads
   (ii) surfaced roads and unsurfaced roads.
3. Write short notes on
   (i) Nagpur Road Plan
   (ii) Bombay Road Plan and
   (iii) Lucknow Road Plan.
4. Distinguish between
   (i) National Highways and State Highways
   (ii) Major District Roads and Other District Roads.
5. Write short notes on
   (i) Expressways
   (ii) National Highways
   (iii) Village Roads
   (iv) Urban Roads.
6. Give the figure to show various components of a carriageway and explain the function of each component.
7. Write short notes on
   (i) Shoulders
   (ii) Kerbs
   (iii) Traffic separators.
8. Give typical cross-sections of
   (i) Divided highway in urban areas
   (ii) MDR in cutting
   (iii) MDR in embankment.

Unit I

CHAPTER 3

Bridges and Dams

Bridge is a structure which provides passage over the obstacles like valley, river, road or railway track without closing the way underneath. The passage required may be for pedestrian, road or for railway track. Initially fallen trees, and wooden logs were used as bridges. With the development of civilization bridge engineering has also developed as a result of which we see many beautiful bridges with longer spans built for roads and railways. In this chapter different types of bridges are discussed with simple sketches.

Dam is a hydraulic structure built across the flow of water to impound water so as to create reservoir. In this chapter different types of dams are briefly explained with simple sketch.

3.1 TYPES OF BRIDGES

Getting ideas from fallen trees and monkeys jumping from one tree to another trees the development of bridge engineering started along with civilization. Construction of wooden bridges date back to 2650 BC in Eqypt. Construction of *setu samudra* during the period of Ramayana is an example of bridge construction in India. Examples of bridge constructions with masonry structures date back to 100 to 500 BC in Persia, Greece, Rome, Egypt, China and India. With improvements with material technology, analysis and design techniques many variety of bridges are built throughout the world. Growing need for large span bridges has also contributed to different types of bridges. Bridges can be classified into various types depending upon the following factors:

1. **Purpose.** Under this heading bridges can be classified as road bridges, railway bridges, foot bridges, aquaduct (for carrying canal water across valley), viaduct (for taking roads across dry valleys).
2. **Alignment.** If the bridge is at right angles to the obstacle it is termed as square bridge. If it is at some other angle the bridge is termed as skew bridge (ref Fig. 3.1)

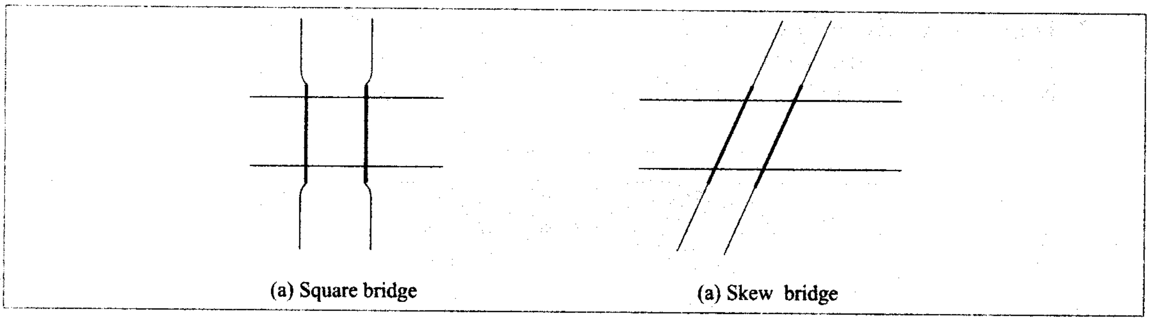


Fig. 3.1

3. **Nature of life.** Under this bridges may be classified as permanent and temporary. Permanent bridges are seen along highways and railways which are built with masonry, R.C.C. or steel with strong foundation. Temporary bridges are built during military operations or during project execution. They are meant for dismantling after the purpose of construction is satisfied.
4. **Span.** Based on the span of the bridges, they may be classified as culverts (less than 8 m), minor bridges (span 8 to 30 m), major bridges (span 30 to 120 m) and long span bridges (span more than 120 m).
5. **Position of high flood level.** Bridges may be classified as submersible and non-submersible. In submersible bridges flow of water above the bridge deck is permitted in heavy rainy season. They are built for roads with lesser importance.
6. **Fixed or movable.** Normally fixed bridges are built. Movable bridges are built across navigable channels so as to avoid obstacles to navigation. Movable bridges may be further classified as (a) swing bridges (b) lift bridges (c) bascule bridges. In case of bascule bridge, the entire superstructure is rotated in vertical plane to 70 to 80°. Suitable hinges and counter weights are provided for easy operations. Fig. 3.2 shows typical bascule bridges.

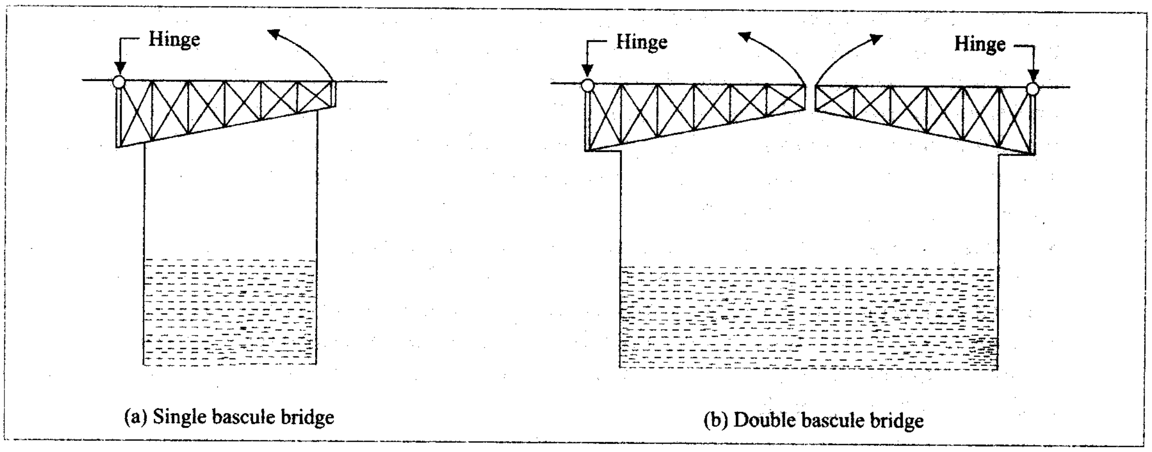


Fig. 3.2 Bascule bridges

7. **Location of bridge floor.** Under this bridge can be classified as deck, semi-through or through type depending upon whether the bridge floor is on top, intermediate or at bottom level of super structure. Fig. 3.3 show such bridges.

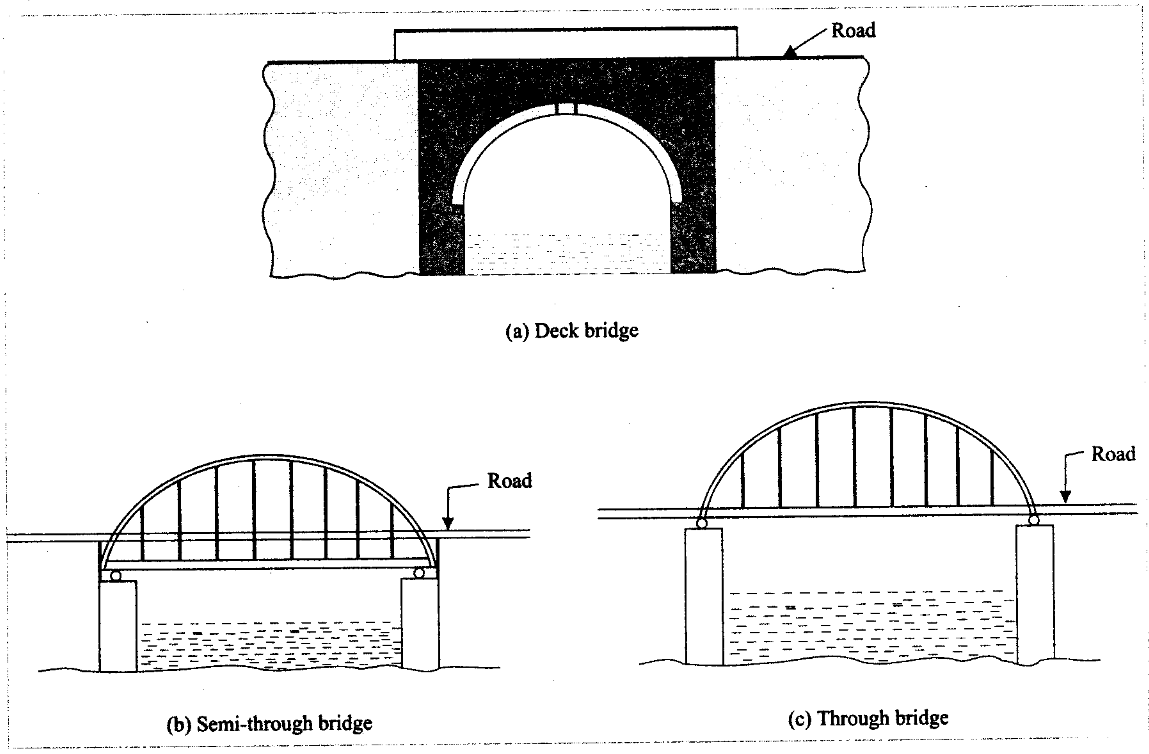


Fig. 3.3 Classification based on location of bridge floor

8. **Superstructure.** Based on superstructure the bridges may be classified as portal frame bridges, truss bridges, cantilever bridges, arch bridges and suspension bridges. Typical portal frame bridge is shown in Fig 3.4(a) and the typical truss bridge is shown in Fig. 3.4(b). The bascule bridge shown in Fig 3.2(b) is an example of double cantilever bridge while deck bridge shown in Fig. 3.3(a) is an example of masonry arch bridge. Fig. 3.4(c) shows the sketch of a suspension bridge. Laxmana zoola at Rishikesh is an example of steel suspension foot bridge while Howrah bridge is an example of suspension bridge for heavy traffic.
9. **Materials.** Based on materials used for the construction of bridges they may be classified as timber bridges, masonry bridges, R.C.C. bridges, steel bridges and prestressed concrete bridges. Masonry bridges are usually arch bridges (Fig 3.3a). Timber, R.C.C. and steel girder may be of girder, arch, contilever, or trussed type. Prestressed concrete bridges are usually girder type or box type. Fig. 3.5 shows typical girder and box type bridges.
10. **Type of connection.** Under this steel bridges may be classified as riveted or welded bridges.

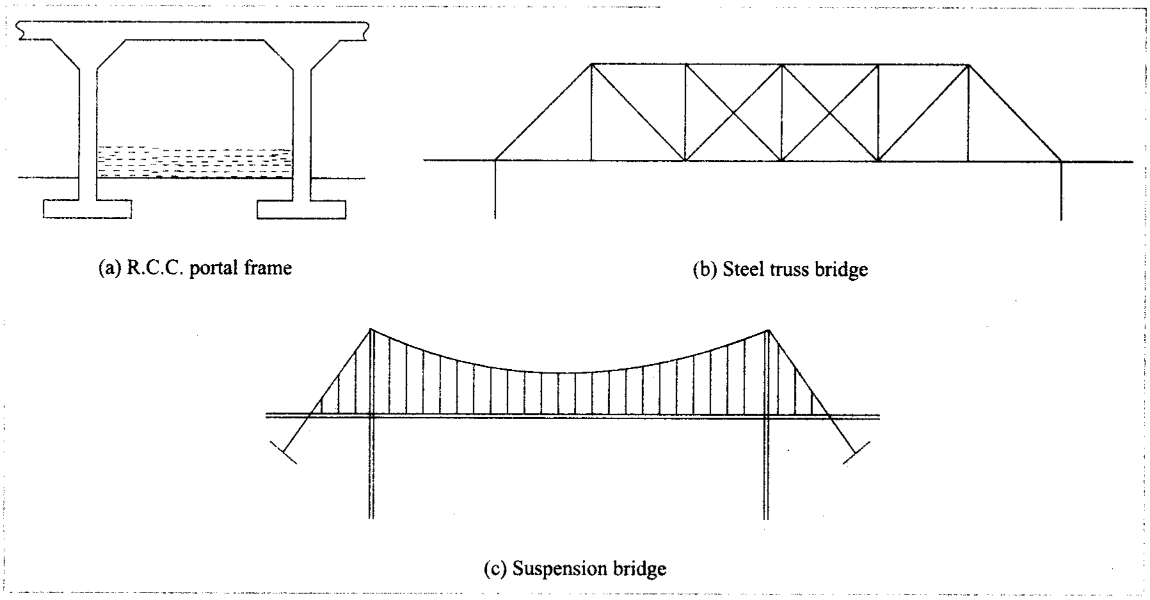


Fig. 3.4 Different types of superstructures

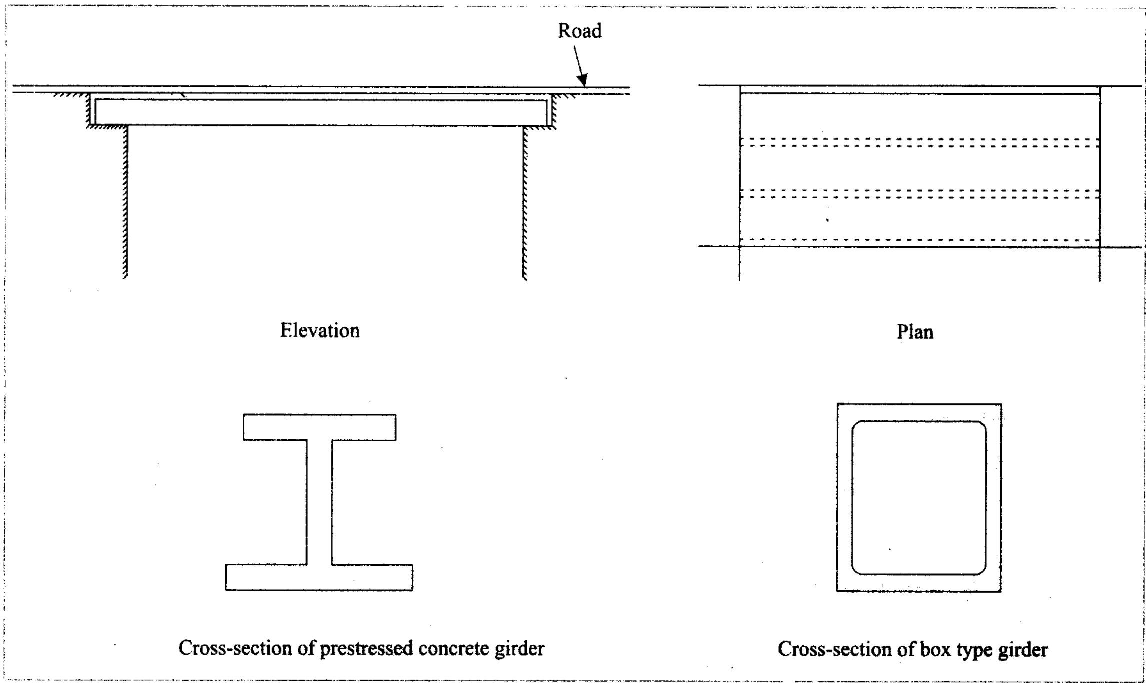


Fig. 3.5 Girder and box type bridges

3.2 TYPES OF DAMS

Dams may be classified on the following basis

(i) Function
(ii) Hydraulic Design
(iii) Structural Behaviour and
(iv) Materials of construction.

3.2.1 Classification Based on Function

Dams are built for various purposes. Based on the purpose the dams have to satisfy, they are classified into the following categories:

(a) **Coffer dam.** During actual dam construction, the dam site is to be kept dry. Hence temporary dams are constructed to divert the stream/river. These temporary dams are known as coffer dams.

(b) **Diversion Dams.** These are the structures built across the river to divert part or whole of water into channels. They are also known as diversion weir.

(c) **Debris Dams.** Dams constructed to retain the debris like sand, silt, gravel and wood flowing with water in the rivers are called as debris dams.

(d) **Detention Dam.** These are the dams primarily constructed to temporarily detain flood waters of a river and to gradually release the stored water at controlled rate so that down stream side is protected from flood havacs.

(e) **Storage Dam.** These are the dams built to store the water to create resorvoir. The stored water is used throughout the year for water supply and/or for irrigation. Such dams may serve as detention dam to control floods. But these dams are primarily built to store the water. Krishnaraj Sagar dam, Tunga Bhadra dam (Hospet), Hidkal dam and Narayanpur dams are some of the famous dams of Karnataka.

3.2.2 Classification Based on Hydraulic Design

Based on hydraulic design dams may be classified as

(a) Overflow dams
(b) Non-overflow dams
(c) Partially overflow dams.

Overflow dams. An overflow dam is constructed with a crest to permit overflow of surplus water. A typical cross-section of such dam is shown in Fig. 3.6.

Non-overflow dams. A non-overflow dam is the one which will not allow overflow of water on it.

In most of the cases part of a dam is designed to permit overflow and part of it is non-overflow portion.

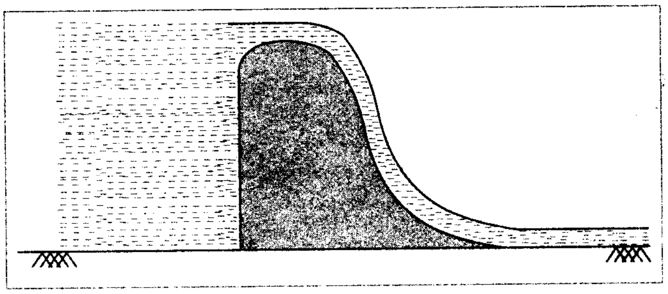


Fig. 3.6 Overflow type gravity dam

3.2.3 Classification Based on Structural Behaviour

On the basis of structural behaviour, the dams may be classified as

(a) Gravity dams
(b) Arch dams
(c) Buttress dams and
(d) Embankment dams.

(a) Gravity dams. The dams which resist the forces exerted by stored water by its self weight are called as gravity dams. They are massive in construction. They are built with concrete or mosonry. They may be solid or may have an inspection gallary within. Figure 3.7 shows cross-section of such dams.

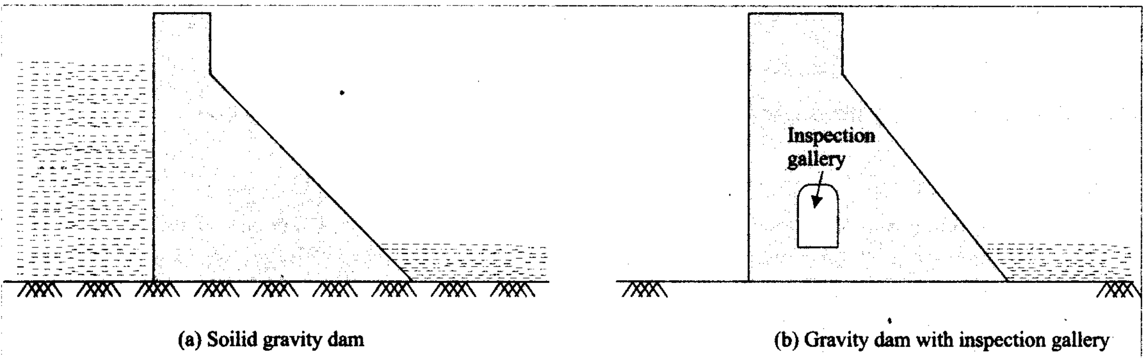


Fig 3.7 Gravity dams

(b) Arch dam. An arch dam is curved in plan. It is built with masonry or concrete. The hydraulic pressure from the reservoir is resisted by arch action. Only arch dam in India is the Iddukki dam. Fig. 3.8 shows a typical arch dam.

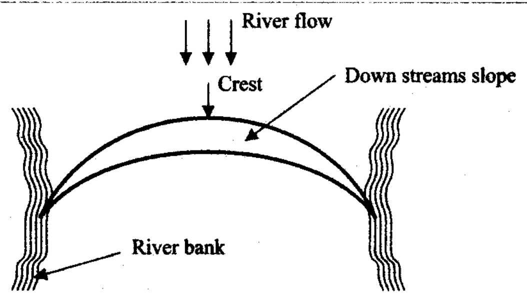


Fig. 3.8 Plan of typical arch dam

(c) Buttress dam. In buttress dams usually reinforced concrete arch slab retains the water. These arches are supported by concrete or masonry buttresses. Typical cross-section of this type of dams are shown in Fig. 3.9.

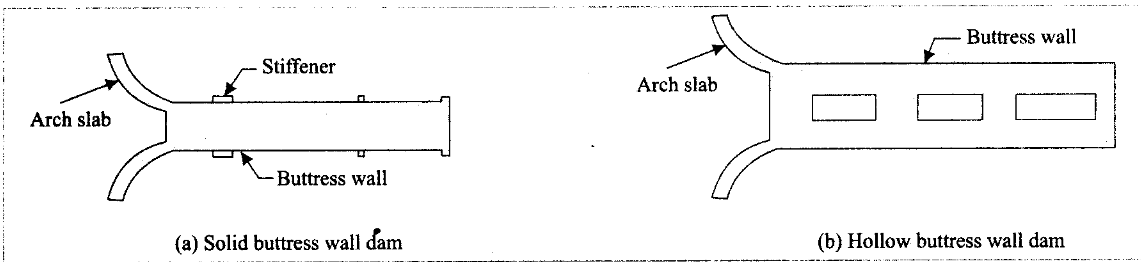


Fig. 3.9 Buttress dam (Plan veiw)

(d) Embankment dam. Embankment dams resist the hydraulic forces mainly by shear. They are made up of non-rigid materials like earth and rock-fill.

3.2.4 Classification Based on Materials of Construction

Material of construction used may be grouped into the following two categories:

(i) Rigid

(ii) Non-rigid

(i) **Rigid dam.** Masonry, concrete, steel or timber dams may be called as rigid dams. Masonry and concrete dams of great height can be built. Bhakra dam is a concrete dam of height 226 m. Timber and steel dams are constructed only for small heights. Now a days they are rarely built. Gravity dams shown in Fig. 3.7, and buttress dams shown in Fig. 3.8 and 3.9 are rigid dams.

(ii) **Non-rigid dams.** Non-rigid materials used for construction of dams are earth, rock fill, tailings etc. Combination of these materials may be used to get impervious dams with sufficient strength. In this type of composite dams upstream side consists of earth fill with stone pitching while down stream side consists of rock fill or sand and gravel fill. They are provided with impervious core which may be by using impervious soil or by using stone masonry. Typical cross sections of such dams are shown in Fig. 3.10

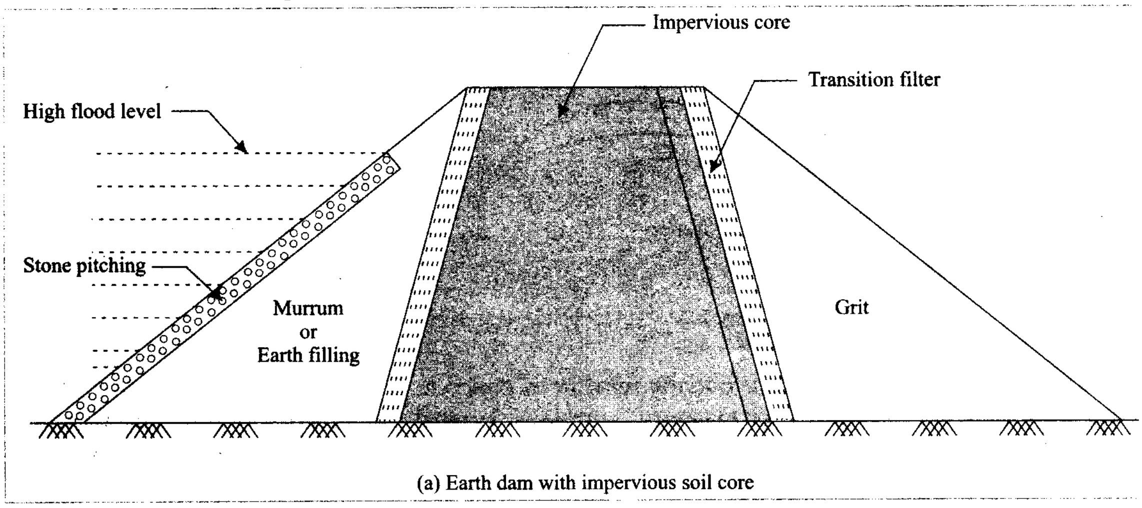
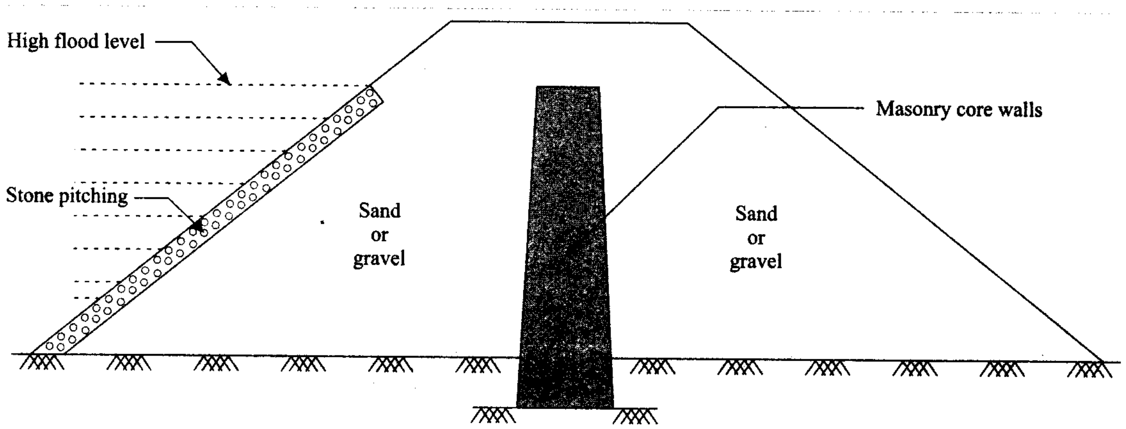


Fig. 3.10 Non-rigid dams (*Contd.*)



(b) Earth dam with masonry core wall

Fig 3.10 Non-rigid dams

Questions

1. Define bridges and dams.
2. Explain the terms: Skew bridges, culverts bascule bridge.
3. Explain the terms: Deck, semi through and through bridges. Give suitable sketches for each one of them.
4. Explain the terms: Coffer dams, diversion dams, detention dams and storage dams.
5. Explain with sketches overflow and non-overflow dams.
6. Give neat sketches of a gravity dam and a buttress dam.
7. Distinguish between rigid and non-rigid material dams. Give typical cross-section of a non-rigid material dam.
8. Classify dams according to their structural behaviour.

Unit II

CHAPTER 4

Introduction to Engineering Mechanics

The state of rest and state of motion of the bodies under the action of different forces have engaged the attention of philosophers, mathematicians and scientists for many centuries. The branch of physical science that deals with the state of rest or the state of motion is termed as Mechanics. Starting from the analysis of rigid bodies under gravitational force and simple applied forces the mechanics has grown to the analysis of robots, aircrafts, space crafts etc. under dynamic forces, atmospheric forces, temperature forces etc.

Archemedes (287-212 BC), Galileo (1564-1642), Sir Issac Newton (1642-1727) and Einstein (1878-1955) have contributed a lot to the development of mechanics. Contributions by Varignon, Euler, D'Alembert are also substantial. The mechanics developed by these researchers may be grouped as:

(i) Classical mechanics (Newtonian mechanics)

(ii) Relativistic mechanics

(iii) Quantum mechanics/wave mechanics

Sir Issac Newton, the principal architect of mechanics, consolidated the philosophy and experimental findings developed around the state of rest and state of motion of the bodies and put forth them in the form of three laws of motion as well as the law of gravitation. The mechanics based on these laws is called Classical mechanics or Newtonian mechanics.

Albert Einstein proved that Newtonian mechanics fails to explain the behaviour of high speed (speed of light) bodies. He put forth the theory of Relativistic Mechanics.

Schrodinger (1887-1961) and Broglie (1892-1965) showed that Newtonian mechanics fails to explain the behaviour of particles when atomic distances are concerned. They put forth the theory of Quantum Mechanics.

Engineers are keen to use the laws of mechanics to actual field problems. Application of laws of mechanics to field problems is termed as Engineering Mechanics. For all the problems between atomic distances and high speed distances Classical/Newtonian mechanics has stood the test of time and hence that is the mechanics used by Engineers. Hence in this book Newtonian Mechanics is used for the analysis of engineering problems.

4.1 BASIC IDEALIZATIONS

A number of ideal conditions are assumed to exist while applying the principles of mechanics to practical problems. In fact without such assumptions it is not possible to arrive at practical solutions. The experience of many centuries has shown that such assumptions/idealizations do not bring down the accuracy of the results of analysis below the tolerable level required by engineers to deal with practical problems. The following basic idealizations are made in engineering mechanics:

- Particle
- Continuum
- Rigid body
- Point force

A **particle** may be defined as an object which has only mass but no size. Such a body cannot exit theoretically. However, while dealing with problems involving distances considerably larger when compared to the size of the body, the size of the body may be neglected without sacrificing accuracy. The examples are

- A bomber aeroplane is a particle for a gunner operating from the ground.
- A ship in mid-sea is a particle while studying its relative motion from a control tower in a port.
- In the study of movement of the earth in the celestial sphere, earth is treated as a particle.

A body consists of several particles. It is a well known fact that each particle can be subdivided into molecules, atoms and electrons. It is not possible to solve any engineering problem by treating a body as a conglomeration of such discrete particles. The body is assumed to consist of a continuous distribution of matter. In other words, the body is treated as a **continuum**. The concept of continuum of bodies enables us to simplify the problems in engineering mechanics.

A **rigid body** may be defined as a body in which the relative positions of any two particles do not change under the action of the forces. For example, consider the body shown in Fig. 4.1. Many engineering problems are solved by assuming that the distance between A and B [Fig. 4.1(a)] is the same as between A' and B' [Fig. 4.1(b)] where A' and B' are the altered positions of the two particles A and B on the body after the application of the forces F_1, F_2 and F_3.

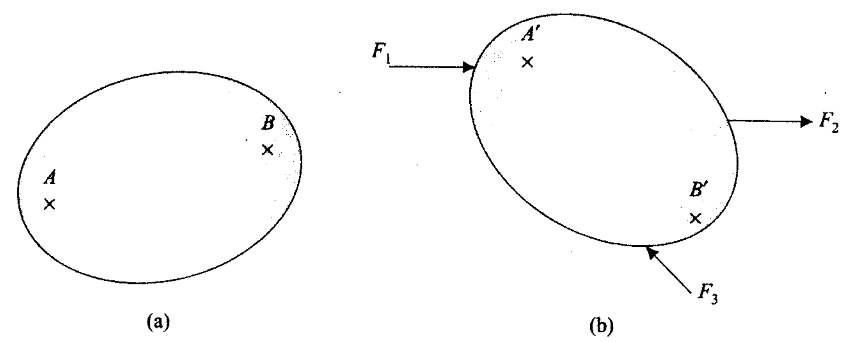


Fig. 4.1

Point force is yet another idealization very commonly used in engineering mechanics. In Fig. 4.2, the weight of the man standing on the ladder is shown as a force applied at the point C. Actually a man cannot apply his weight through a single point. There is certain area of contact. This contact area is ignored compared to other dimensions involved in the problem. Hence the force is treated as if it acts through the point C. Not much accuracy is lost by treating it as a point force and thereby simplifying the problem.

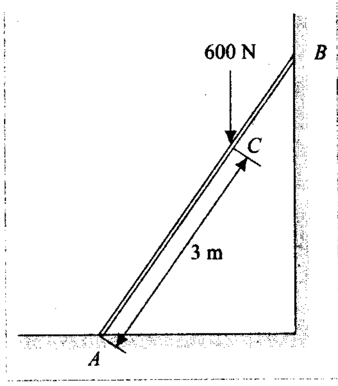


Fig. 4.2

There are many similar idealizations assumed in engineering mechanics to find the solutions for practical problems, however, without losing desired accuracy. They relate to the support conditions and the system of forces. They are explained in the text along with the relevant topics.

4.2 NEWTON'S LAWS OF MOTION

Newtonian mechanics is based on the following fundamental laws proposed by Sir Issac Newton:

- Newton's first law
- Newton's second law
- Newton's third law
- Newton's law of gravitation

They are defined and explained in this article.

Newton's First Law

It states that every body continues in its state of rest or of uniform motion in a straight line unless it is compelled by an external agency acting on it. This leads to the definition of force as *the external agency which changes or tends to change the state of rest or uniform linear motion of the body.*

Newton's Second Law

It states that *the rate of change of momentum of a body is directly proportional to the impressed force and it takes place in the direction of the force acting on it.* Thus, according to this law,

Force ∝ rate of change of momentum

But momentum = mass × velocity

As mass does not change,

Force = mass × rate of change of velocity

i.e., Force ∝ mass × acceleration

$$F \propto m \times a \qquad \text{Eqn. (4.1)}$$

where 'F' is force, 'm' is mass and 'a' is acceleration.

Newton's Third Law

It states that *for every action there is an equal and opposite reaction.* Consider the two bodies in contact with each other. Let one body apply a force F on another. According to this law the second body develops a reactive force R which is equal in magnitude to force F and acts in the line same as F but in the opposite direction. Figure 4.3 shows the action of the ball and the reaction from the floor. In Figure 4.4, the actions of the ladder on the wall and the floor and the reactions from wall and floor are shown.

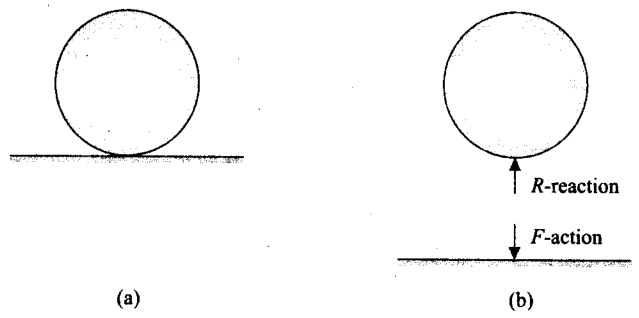


Fig. 4.3

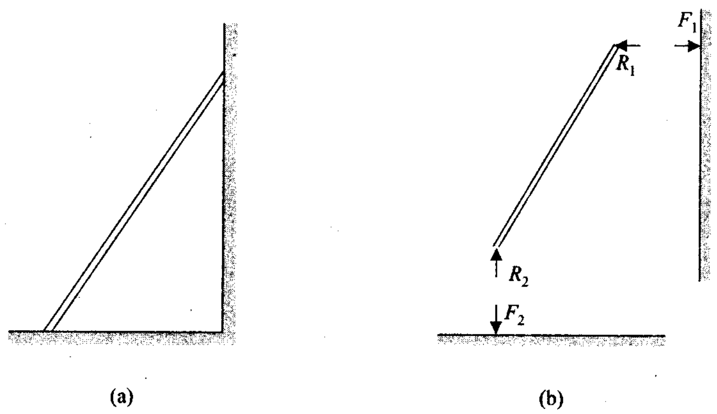


Fig. 4.4

Newton's Law of Gravitation

Every body attracts other body. The force of attraction between any two bodies is directly proportional to their masses and inversely proportional to the square of the distance between them. According to this law the force of attraction between the bodies of mass m_1 and m_2 at distance 'd' as shown in Fig. 4.5 is

$$F = G\,\frac{m_1 m_2}{d^2} \qquad \text{Eqn. (4.2)}$$

where G is constant of proportionality and is known as constant of gravitation.

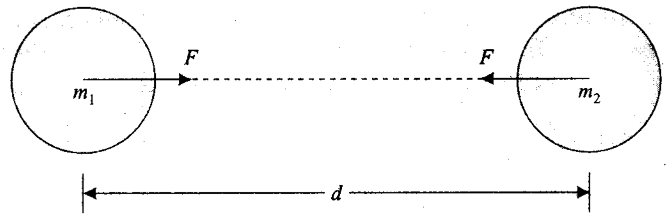


Fig. 4.5

4.3 INTRODUCTION TO SI UNITS

Length (L), mass (M) and time (S) are the fundamental units in mechanics. The unit of all other quantities may be expressed in terms of these basic units. The three commonly used systems in engineering are

- Metre – Kilogramme – Second (MKS) system
- Centimetre – Gramme – Second (CGS) system, and
- Foot – Pound – Second (FPS) system.

The units of length, mass and time used in the system are used to name the systems. Using these basic units, the units for other quantities can be found. For example, in MKS the units for the various quantities are as shown below:

Table 4.1

Quantity	*Unit*	*Notation*
Area	square metre	m^2
Volume	cubic metre	m^3
Velocity	metre per second	m/sec
Acceleration	metre per second per second	m/sec^2

Unit of Forces

Presently the whole world is in the process of switching over to *SI system of units.* SI stands for System Internationale *d'* units or International System of units. As in MKS system, in SI system also the fundamental units are metre for length, kilogramme for mass and second for time. The difference between MKS and SI system arises mainly in selecting the unit of force. From Eqn. (1.3), we have

$$\text{Force} \propto \text{Mass} \times \text{Acceleration} = k \times \text{Mass} \times \text{Acceleration} \qquad \text{Eqn. (4.3)}$$

In SI system, unit of force is defined as that force which causes 1 kg mass to move with an acceleration of 1 m/sec^2 and is termed as 1 newton. Hence the constant of proportionality k becomes unity. Unit of force can be derived from Eqn. 4.3 as

$$\text{Unit of Force} = \text{kg} \times \text{m/sec}^2$$
$$= \text{kg - m/sec}^2$$

In MKS, the unit of force is defined as that force which makes a mass of 1 kg to move with gravitational acceleration 'g' m/sec^2. This unit of force is called kilogramme weight or kg-wt.

Gravitational acceleration is 9.81 m/sec^2 near the earth surface. In all the problems encountered in engineering mechanics the variation in gravitational acceleration is negligible and may be taken as 9.81 m/sec^2. Hence the constant of proportionality in equation 4.3 is 9.81, which means

$$1 \text{ kg-wt} = 9.81 \text{ newton} \qquad \text{Eqn. (4.4)}$$

It may be noted that in public usage, kg-wt force is called as kg only.

Unit of Constant of Gravitation

From equation 4.2

$$F = G\frac{m_1 m_2}{d^2}$$

or

$$G = \frac{Fd^2}{m_1 m_2}$$

$$\text{Unit of } G = \frac{\text{N} \times \text{m}^2}{\text{kg} \times \text{kg}} = \text{Nm}^2/\text{kg}^2$$

It has been proved by experimental results that the value of $G = 6.673 \times 10^{-11}$ Nm^2/kg^2. Thus if two bodies one of mass 10 kg and the other of mass 5 kg are at a distance of 1 m, they exert a force

$$F = \frac{6.673 \times 10^{-11} \times 10 \times 5}{1^2} = 33.365 \times 10^{-10} \text{ N}$$

on each other.

Now let us find the force acting between 1 kg-mass near earth surface and the earth. Earth has a radius of 6371×10^3 m and has a mass of 5.96506×10^{24} kg. Hence the force between the two bodies is

$$= \frac{6.673 \times 10^{-11} \times 1 \times 5.96506 \times 10^{24}}{(6371 \times 10^3)^2} = 9.80665 \text{ N.}$$

In common usage we call the force exerted by earth on a body as *weight of the body*. Thus weight of 1 kg mass on earth surface is 9.80665 N, which is approximated as 9.81 N for all practical problems. Compared to this force the force exerted by two bodies near earth surface is negligible as may be seen from the example of 10 kg and 5 kg mass bodies.

Denoting the weight of the body by W, from expression 4.2, we get

$$W = \frac{GmM_e}{r^2}$$

where m is the mass of the body

M_e is the mass of the earth, and

r is the radius of the earth

Denoting $\frac{G M_e}{r^2}$ by g, we get

$$W = mg = 9.81 \text{ m} \qquad \text{Eqn. (4.5)}$$

Unit of g can be obtained as follows:

$$g = \frac{G M_e}{r^2}$$

$$\text{Unit of } g = \frac{\text{Nm}^2}{(\text{kg})^2} \times \frac{\text{kg}}{\text{m}^2} = \frac{\text{N}}{\text{kg}}$$

As unit of newton-force is kg-m/sec^2, we get

$$\text{Unit of } g = \frac{\text{kg m/sec}^2}{\text{kg}} = \text{m/sec}^2$$

Hence g may be called as acceleration due to gravity. Any body falling freely near earth surface experiences this acceleration. The value of g is 9.81 m/sec^2 near the earth surface as can be seen from equation 4.5.

The prefixes used in SI system when quantities are too big or too small are shown in Table 4.2.

Table 4.2 Prefixes and Symbols of Multiplying Factors in SI

Multiplying Factor	*Prefix*	*Symbol*
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	K
10^{0}	–	–
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

4.4 ELEMENTS OF A FORCE

From Newton's first law, we defined the force as the agency which tries to change state of rest or state of uniform motion of the body. From Newton's second law of motion we arrived at practical definition of unit force as the force required to produce unit acceleration in a body of unit mass. Thus, 1 newton is the force required to produce an acceleration of 1 m/sec^2 in a body of 1 kg mass. It may be noted that a force is completely specified only when the following four characteristics are specified:

- Magnitude
- Point of application
- Line of action and
- Direction.

In Fig. 4.2, *AB* is a ladder kept against a wall. At point *C*, a person weighing 600 N is standing. The force applied by the person on the ladder has the following characters:

- magnitude is 600 N
- the point of application is at *C* which is 3 m from *A* along the ladder
- the line of action is vertical and
- the direction is downward.

Note that the magnitude of the force is written near the arrow. The line of the arrow shows the line of application and the arrowhead represents the point of application and the direction of the force.

4.5 SYSTEM OF FORCES

When several forces act simultaneously on a body, they constitute a *system of forces*. If all the forces in a system do not lie in a single plane they constitute the *system of forces in space*. If all the forces in a system lie in a single plane, it is called a *coplanar force system*. If the line of action of all the forces in a system pass through a single point, it is called a *concurrent force system*. In a *system*

Table 4.3 System of Forces

Force System	*Characteristics*	*Examples*
Collinear forces	Lines of action of all the forces act along the same line.	Forces on a rope in a tug of war.
Coplanar parallel forces	All forces are parallel to each other and lie in a single plane.	System of forces acting on a beam subjected to vertical loads (including reactions).
Coplanar-like parallel forces	All forces are parallel to each other, lie in a single plane and are acting in the same direction.	Weight of a stationary train on a rail when the track is straight.
Coplanar concurrent forces	Lines of action of all forces pass through a single point and forces lie in the same plane.	Forces on a rod resting against a wall.
Coplanar non-concurrent forces	All forces do not meet at a point, but lie in a single plane.	Forces on a ladder resting against a wall when a person stands on a rung which is not at its centre of gravity.
Non-coplanar parallel forces	All the forces are parallel to each other, but not in same plane.	The weight of benches in a classroom.
Non-coplanar concurrent forces	All forces do not lie in the same plane, but their lines of action pass through a single point.	A tripod carrying a camera.
Non-coplanar nonconcurrent forces	All forces do not lie in the same plane and their lines of action do not pass through a single point.	Forces acting on a moving bus.

of parallel forces all the forces are parallel to each other. If the lines of action of all the forces lie along a single line then it is called a *collinear force system*. Various systems of forces, their characteristics and examples are given in Table 4.3 and are shown in Fig. 4.6.

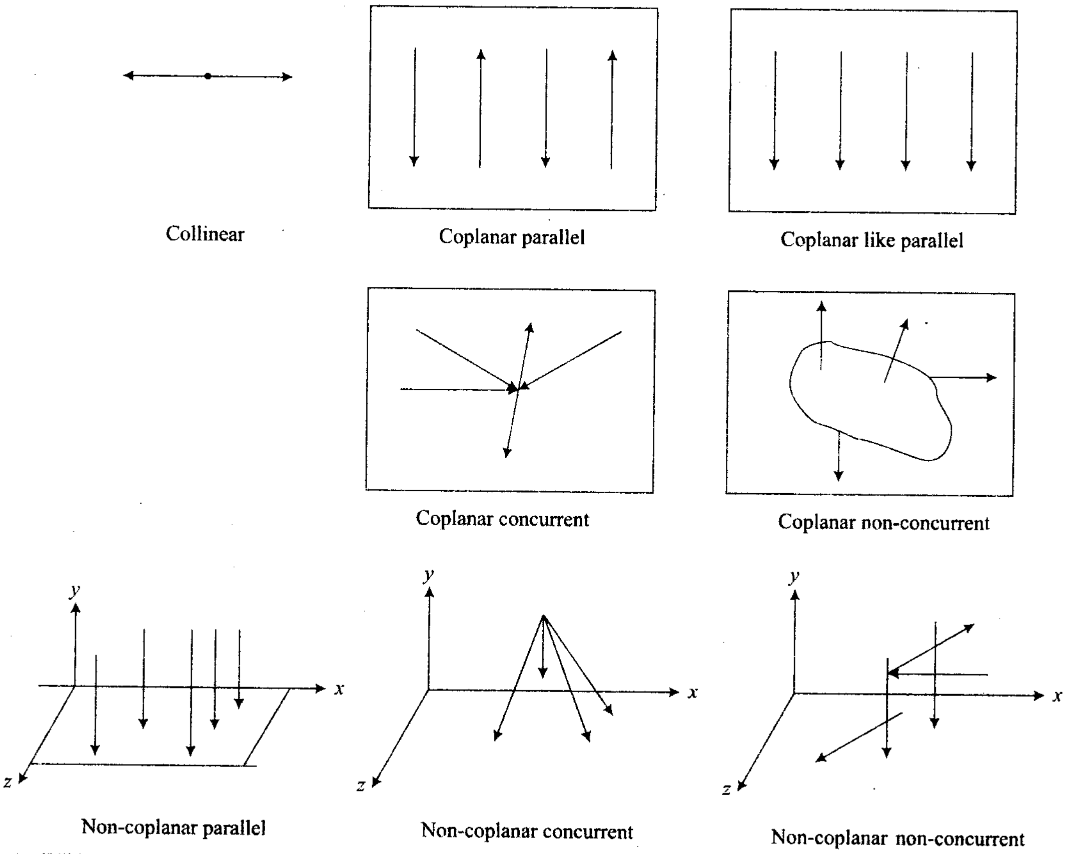


Fig. 4.6

4.6 VECTORS

Various quantities used in engineering mechanics may be grouped into scalars and vectors. A quantity is said to be *scalar* if it is completely defined by its magnitude alone. Examples of scalars are length, area, time and mass.

A quantity is said to be *vector* if it is completely defined only when its magnitude as well as direction are specified. Hence force is a vector. The other examples of vector are velocity, acceleration, momentum etc.

4.7 PRINCIPLES OF PHYSICAL INDEPENDENCE SUPERPOSITION AND TRANSMISSIBILITY OF FORCES

The following three principles are used is solving engineering mechanics problems:

1. Principles of physical independence of forces
2. Principles of superposition of forces
3. Principles of transmissibility of forces

These principles are briefly explained bellow:

4.7.1 Principles of Physical Independence of Forces

It states that *the action of a force on a body is not affected by the action of any other force on the body.*

4.7.2 Principles of Superposition of Force

It states that net effect of a system of forces on a body is same as the combined effect of individual forces acting on the body. Since a system of forces in equilibrium do not have any effect on a rigid body this principle is stated in the following form also: *'The effect of a given system of forces on a rigid body is not changed by adding or subtracting another system of forces in equilibrium'*

4.7.3. Principle of Transmissibility of Forces

According to this principle *the state of rest or motion of the body is unaltered if a force acting on a rigid body is replaced by another force of the same magnitude and direction but acting anywhere on the body along the line of action of the replaced force.*

Let F be the force acting on a rigid body at point A as shown in Fig. 4.7(a). According to the law of transmissibility this force has the same effect on the state of the body as the force F applied at point B.

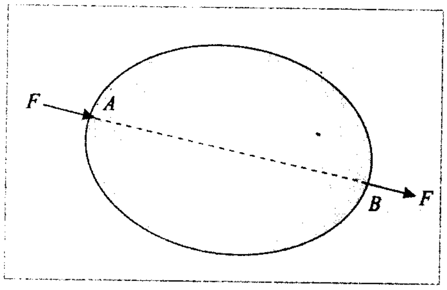


Fig. 4.7(a)

In using the principle of transmissibility of forces it should be noted carefully that it is applicable only if the body can be treated as rigid. In this book, the engineering mechanics is restricted to study of state of rest of rigid bodies and hence this principle is frequently used. Same thing cannot be done in the subject strength of materials, where the bodies are treated as deformable and internal forces in the body are studied.

Consider the rigid body shown in Fig. 4.7(b). It is subjected to a force F at A. B is another point on the line of action of the force. It is obvious from the law of superposition that if two equal and opposite forces of magnitude F are applied at B along the line of action [Ref. Fig. 4.7(c)], the effect of given force on the body is not altered. Force F at A and opposite force F and B form a system of forces in equilibrium. If these two forces are subtracted from the system, the resultant system is as shown in Fig. 4.7(d). Looking at the system of forces in Figs. 4.7(b) and (d), we can conclude that the law of transmissibility is proved.

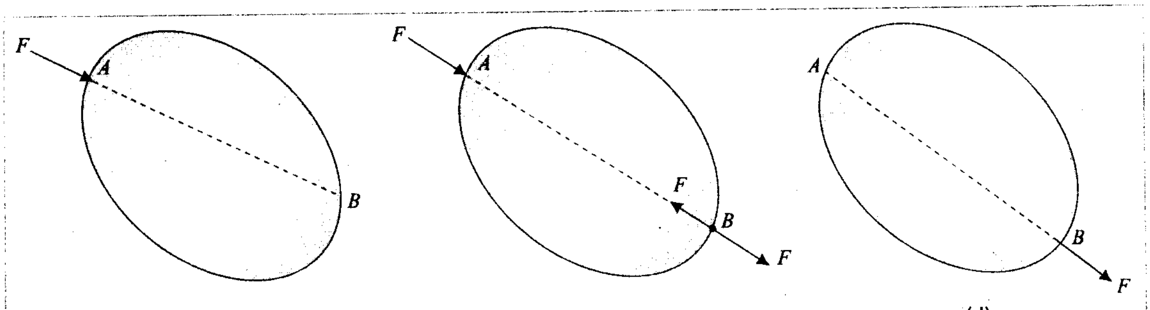


Fig. 4.7

4.8 MOMENT OF A FORCE

Moment of a force about a point is the measure of its rotational effect. **Moment** is defined as *the product of the magnitude of force and the perpendicular distance of the point from the line of action of the force.* The point about which the moment is considered is called **moment centre** and the perpendicular distance of the point from the line of action of the force is called **moment arm.** Referring to Fig. 4.8 if d_1 is the perpendicular distance of point 1 from the line of action of force F, the moment of F about point 1 is given by

$$M_1 = Fd_1$$

Similarly, moment about point 2 is given by

$$M_2 = Fd_2$$

If the moment centre 3 lies on the line of action of the force F, the moment arm is zero and hence

$$M_3 = F \times 0 = 0$$

Thus, it may be noted that if a point lies on the line of action of a force, the moment of the force about that point is zero.

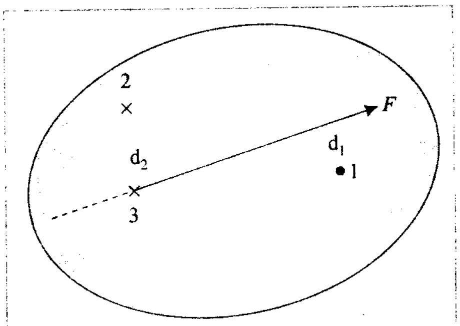


Fig. 4.8

The moment of a force has got direction also. In Fig. 4.8, it may be noted that M_1 is clockwise and M_2 is anticlockwise. To find the direction of the moment, imagine that the force is connected to the point by a rigid rod pinned at the moment centre and is free to rotate around. The direction of the rotation indicates the direction of the moment.

If the force is taken in newton unit and the distance in millimetre, the unit of moment will be N-mm. Commonly used units of moment in engineering are kN-m, N-m, kN-mm and N-mm.

4.9 COUPLE, MOMENT OF A COUPLE AND ITS CHARACTERISTICS

Two parallel forces equal in magnitude and opposite in direction and separated by a definite distance are said to form a couple. The sum of the forces forming a couple in any direction is zero, which means the translatory effect of the couple is zero.

An interesting property can be observed if we consider rotational effect of a couple about any point. Let the magnitude of the forces forming the couple be F and the perpendicular distance between the two forces be d. Consider the moment of the two forces constituting a couple about point 1 as shown in Fig. 4.9(a). Let the moment be M_1

Then,

$$M_1 = Fd_1 + Fd_2$$
$$= F(d_1 + d_2)$$
$$= Fd$$

Now, consider the moment of the forces about point 2 which is outside the two forces as shown in Fig. 4.9(b). Let M_2 be the moment.

Then,

$$M_2 = Fd_3 - Fd_4$$
$$= F(d_3 - d_4)$$
$$= Fd$$

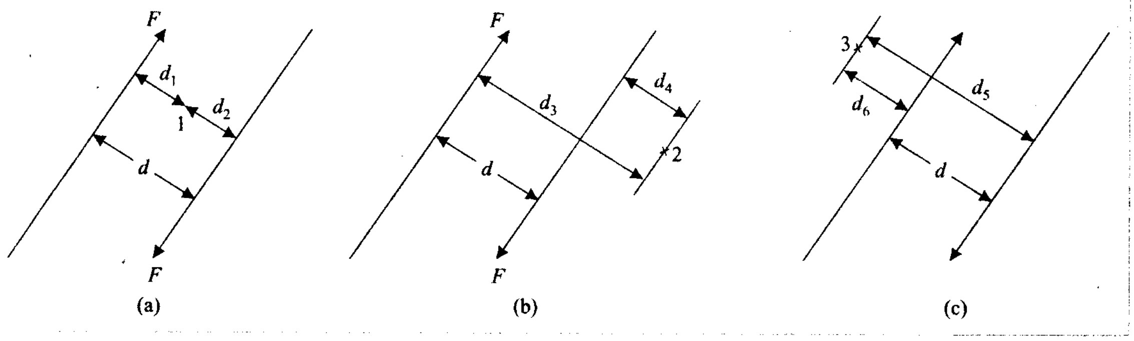


Fig. 4.9

Similarly, it can be seen that $M_3 = Fd$.

Thus, moment of a couple about any point is the same. Now we can list the following characteristics of a couple.

- A couple consists of a pair of equal and opposite parallel forces which are separated by a definite distance;
- The translatory effect of a couple on the body is zero;
- The rotational effect (moment) of a couple about any point is a constant and it is equal to the product of the magnitude of the forces and the perpendicular distance between the two forces.

Since the only effect of a couple is a moment and this moment is the same about any point, the effect of a couple is unchanged if:

- The couple is rotated through any angle;
- The couple is shifted to any other position;
- The couple is replaced by another pair of forces whose rotational effects are the same.

4.10 EQUIVALENT FORCE-COUPLE SYSTEM

It will be advantageous to resolve a force acting at a point on a body into a force acting at some other suitable point on the body and a couple. In Fig. 4.10, F is a force acting on a body at A.

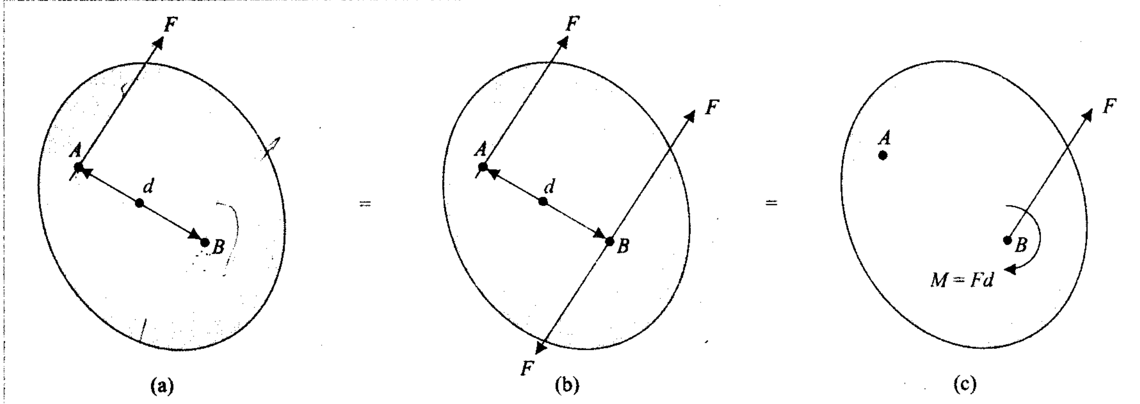


Fig. 4.10

Now, it is possible to show that F at A may be resolved into a force F at B and a moment $M = F \times d$, where d is the perpendicular distance of B from the line of action of F acting through A.

By applying equal and opposite forces P at B the system of forces is not disturbed. Hence the system of forces in Fig. 4.10(b) is the same as the system given in Fig. 4.10(a). Now the original force F acting at A and the opposite force F at B form a couple of magnitude Fd. The system in Fig. 4.10(b) can be replaced by the system shown in Fig. 4.10(c). Thus the given force F acting at point A is replaced by a force F acting at B and a moment equal to Fd.

4.11 RESOLUTION AND COMPOSITION OF FORCES

Resolution of forces is the process of finding a number of component forces which will have the same effect as the given single force. In the previous example we have resolved force F acting at A into its component force F at B and a moment $F \times d$. Process of finding a single force to get the same effect as a number of forces is termed as composition of forces. In the previous article, if F and M are given at B we can locate its equivalent single force as F at A which is at a distance $d = \frac{M}{F}$. Resolution and composition of forces for concurrent force system is explained in Chapter 5 and for nonconcurrent systems in Chapter 6.

4.12 NUMERICAL PROBLEMS ON EQUIVALENT FORCE-COUPLE SYSTEM

Following problems are solved to explain the equivalent force-couple system. Sign conventions used in the problems are

- rightward and upward forces—positive
- clockwise moment—positive.

Example 4.1 Replace the horizontal 600 N force acting on the lever by an equivalent system consisting of a force and a couple at 'O' [Ref. Fig. 4.11].

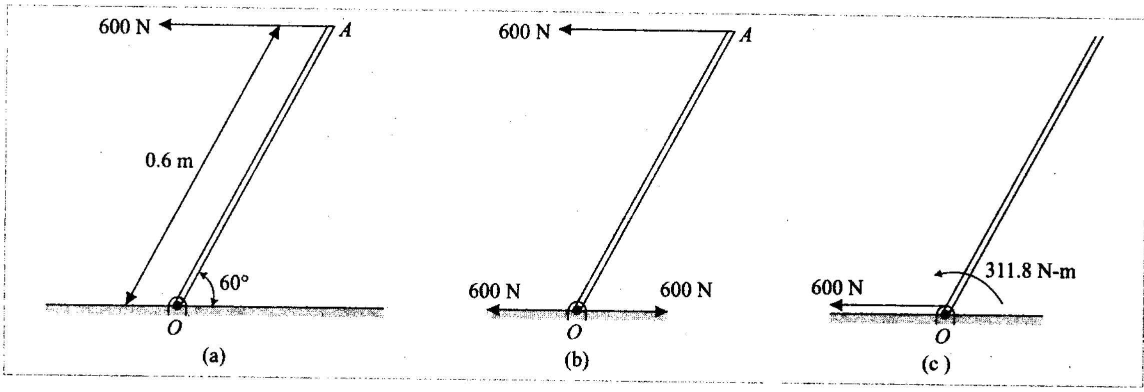


Fig. 4.11

Solution. Applying two equal and opposite 600 N forces at O and then looking for the couple given by the given force and opposite force at O, we find anticlockwise couple at O

$$= 600 \times 0.6 \sin 60 = 311.8 \text{ N-m}$$

Equivalent force at O is 600 N.

Thus the original force of 600 N at A is equivalent to **600 N force** and **311.8 N-m couple** at O as shown in Fig. 4.11(c).

Ans.

Example 4.2 Replace the 50 kN force acting on a concrete column at an eccentricity $e_x = 100$ mm by a force-couple system acting at O (Ref. Fig. 4.12).

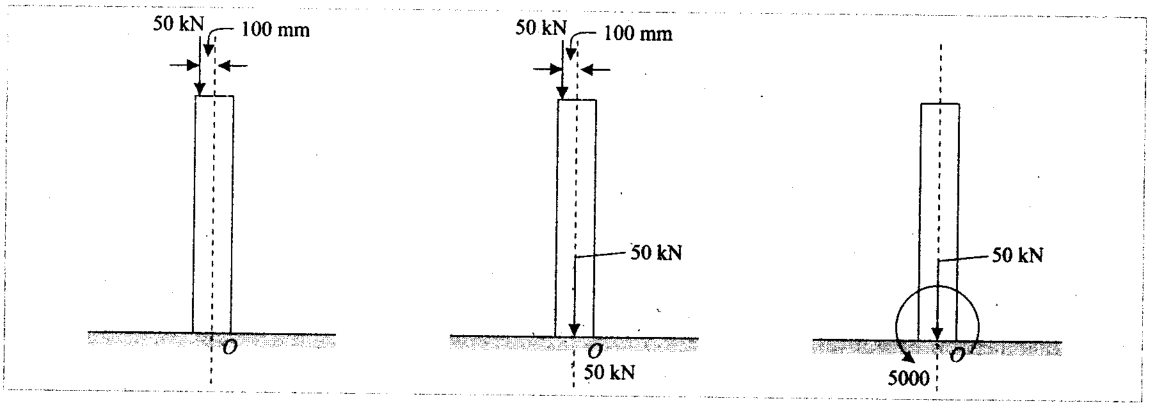


Fig. 4.12

Solution. Applying two equal and opposite forces of 50 kN at 0 in the line of action parallel to given force, we find a 50 kN vertical load is acting downward and a couple of 50 kN separated by 100 mm is also acting. Replacing the couple by couple moment

$$M = 50 \times 100 = 5000 \text{ kN-mm}$$

we find the force-couple system at O is

∴ a force of 50 kN downward

∵ a couple moment of **5000 kN-mm**, clockwise **Ans.**

Example 4.3 The top view of a revolving entrance door is shown in Fig. 4.13. Two persons approach simultaneously and exert forces of equal magnitude as shown in the figure. If the resulting moment about the door pivot axis at O is 20 N-m, determine the magnitude of forces.

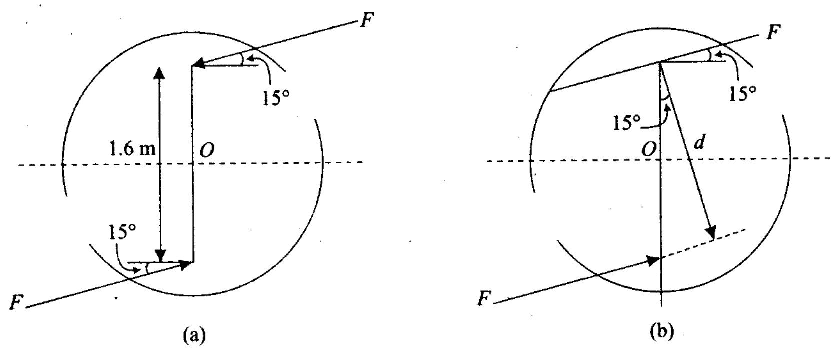


Fig. 4.13

Solution. Let the forces applied be F. The perpendicular distance between the two forces is

$$d = 1.6 \cos 15 = 1.545 \text{ m}$$

∴ $$M = F \times d = F \times 1.545$$

This is given as 20 N-m.

∴ $$1.545\, F = 20$$

or $$\mathbf{F = 12.94 \text{ N}}$$ **Ans.**

Important Definitions and Concepts

1. *Acceleration* is the rate of change of velocity with respect to time.
2. The product of mass and velocity is called *momentum*.
3. A body is said to be treated as *continuum*, if it is assumed to consist of continuous distribution of matter.
4. A body is said to be *rigid*, if the relative positions of any two particles in it do not change under the action of the forces.
5. *Newton's first law* states that every body continues in its state of rest or of uniform motion in a straight line unless it is compelled by an external agency acting on it.
6. *Newton's second law* states that the rate of change of momentum of a body is directly proportional to the impressed force and it takes place in the direction of the force acting on it.

7. *Newton's third law* states for every action there is an equal and opposite reaction.
8. *Newton's law of gravitation* states every body attracts the other body. The force of attraction between any two bodies is directly proportional to their mass and inversely proportional to the square of the distance between them.
9. According to the *law of transmissibility of force,* the state of rest or motion of a rigid body is unaltered if a force acting on a body is replaced by another force of the same magnitude and direction but acting anywhere on the body along the line of action of the replaced force.
10. A quantity is said to be *scalar,* if it is completely defined by its magnitude alone.
11. A quantity is said to be *vector,* if it is completely defined only when its magnitude as well as direction are specified.

Questions

4.1. Explain the following terms as used in Engineering Mechanics.
(i) Continuum
(ii) Rigid body
(iii) Particle

4.2. State and explain Newton's three laws of motion.

4.3. State and explain Newton's law of gravitation.

4.4. State and explain law of transmissibility of forces.

4.5. Explain the term 'Force' and list its characteristics.

4.6. Explain the terms—concurrent and non-concurrent force systems; planar and non-planar system of forces.

Problems for Exercise

4.1 While taking left turn, a driver exerts two 10 N forces on a steering wheel as shown in Fig. 4.14. Determine the moment associated with these forces. Discuss the effect of varying steering wheel diameter d.

[**Ans.** M = 4000 N-mm, anticlockwise. Moment increases with the diameter]

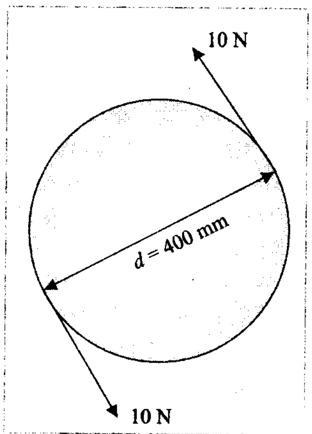


Fig. 4.14

4.2 A door need 7500 N-mm force to open it. Mr. A applies the force at the edge of the shutter which is at a distance of 750 mm from hinge and Mr. B applies it at a distance of 500 mm from the hinge. What forces have they to apply to open the door?

[**Ans.** F_A = 10 N and F_B = 15 N]

Unit II

CHAPTER 5

Composition of Coplanar Concurrent Forces

It is possible to find a single force which will have the same effect as that of a number of forces acting on a body. Such single force is called the Resultant force of the system. The process of finding the resultant force is called composition of forces. In this chapter first parallelogram law, triangle law and polygon law of forces are explained and then general method of finding resultant of concurrent forces is explained. Numerical problems are solved to illustrate the procedure.

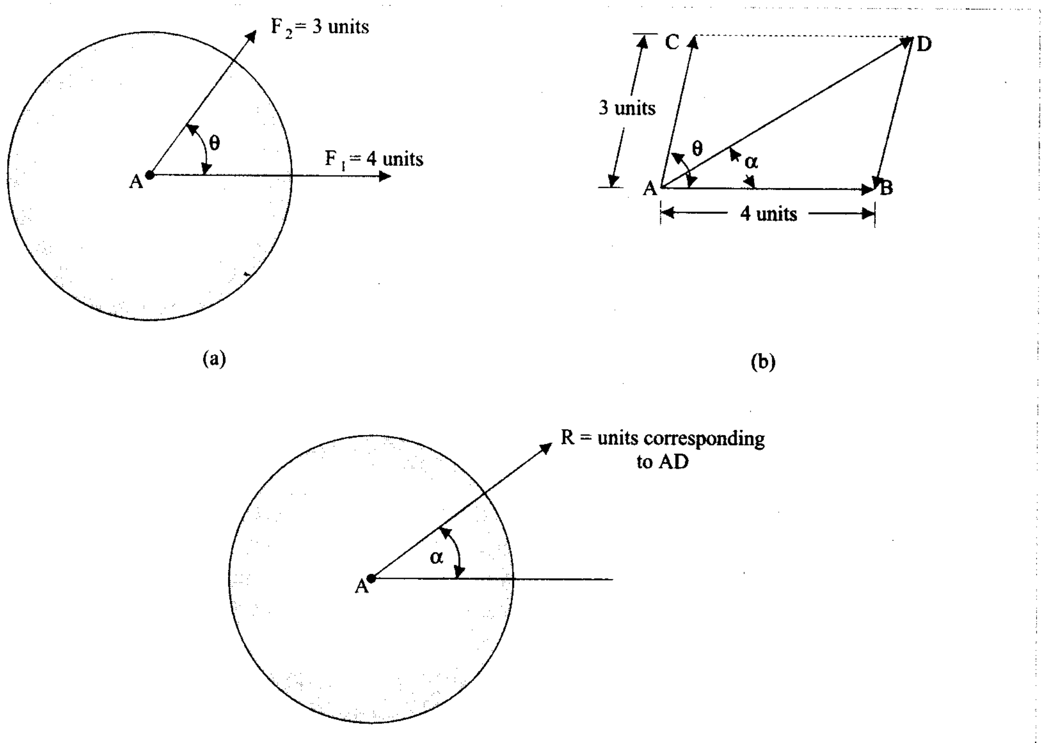
5.1 PARALLELOGRAM LAW OF FORCES

This law is the basic law in mechanics for finding the resultant of concurrent forces. This law was formulated based on experimental results. Though Stevinces employed it in 1586, the credit of presenting it as a law goes to Varignon and Newton (1687). This law states that *if two forces acting simultaneously on a body at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram, their resultant is represented in magnitude and direction by the diagonal of the parallelogram which passes through the point of intersection of the two sides representing the forces.*

In Fig. 5.1, the force F_1 = 4 kN and force F_2 = 3 kN are acting on a body at point A simultaneously. Then to get the resultant of these two forces, parallelogram $ABCD$ is constructed such that AB is equal to 4 units (say 40 mm) and AC is 3 units (say 30 mm). Then according to this law the diagonal AD represents the resultant in direction and magnitude. Thus the resultant of forces F_1 and F_2 on the body is equal to units corresponding to AD (10 mm = 1 kN).

5.2 DERIVED LAWS

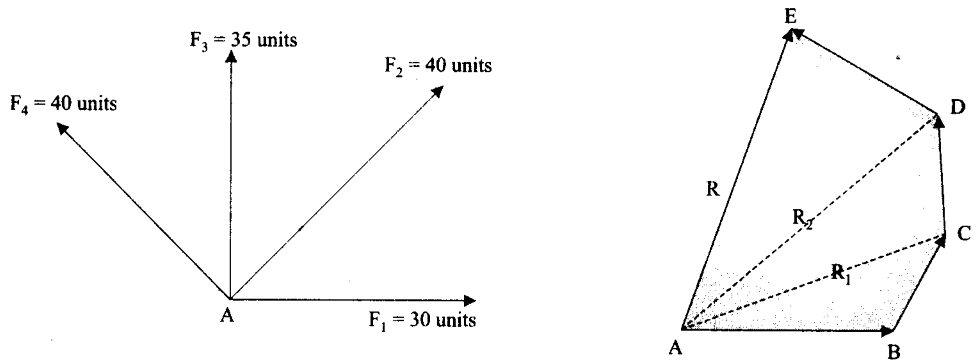
Referring to Fig. 5.1(b), we can get the resultant AD by constructing a triangle ABD. Line AB is drawn to represent F_1 and BD to represent F_2. Then AD should represent the resultant of F_1 and



(a) (b) (c)

F_2. Thus we have derived triangle law of forces. The triangle law of forces may be stated as '*if two forces acting on a body are represented one after another by the sides of a triangle, their resultant is represented by the closing sides of the triangle taken from first point to the last point.*

If more than two concurrent forces are acting on a body, two forces at a time may be combined using triangle law of forces and finally resultant of all forces acting on the body may be obtained. A system of four forces acting on a body is shown in Fig. 5.2(a).



(a) (b)

AB represents force F_1 and BC represents force F_2. Hence according to triangle law of forces AC represents the resultant of forces F_1 and F_2, say R_1.

If CD is drawn to represent F_3, then from triangle law of forces AD represents the resultant of R_1 and F_3. In other words, AD represents the resultant of F_1, F_2 and F_3. Let it be called R_2. On the same line, logic can be extended to conclude that AE represents the resultant of F_1, F_2, F_3 and F_4, if DE represents F_4. Thus the resultant R of the given concurrent forces is represented by the closing line of the polygon $ABCDE$ in the direction AE. Thus we have derived polygon law of forces and it may be stated as, *if a number of concurrent forces acting simultaneously on a body are represented in magnitude and direction by the sides of a polygon, taken in order, then the resultant is represented in magnitude and direction by the closing line of the polygon, taken from the first point to the last point.*

5.3 COMPOSITION OF COPLANAR CONCURRENT TWO FORCE SYSTEM

One can find the resultant of coplanar concurrent force system using parallelogram law or triangle law of forces. The advantage of these methods is they give clear picture of the work being carried out. But these methods being graphical methods they need drawing aids like pencil, scale, drawing sheets, drawing boards etc. Hence engineers prefer to go for the analytical method of finding resultant.

Consider the two forces F_1 and F_2 acting on a particle as shown in Fig. 5.3(a). Let the angle between the two forces be θ. If parallelogram $ABCD$ is constructed as shown in Fig. 5.3(b), with AB representing F_1 and AD representing F_2 to some scale, according to parallelogram law of forces AC represents the resultant R. Drop perpendicular CE to AB.

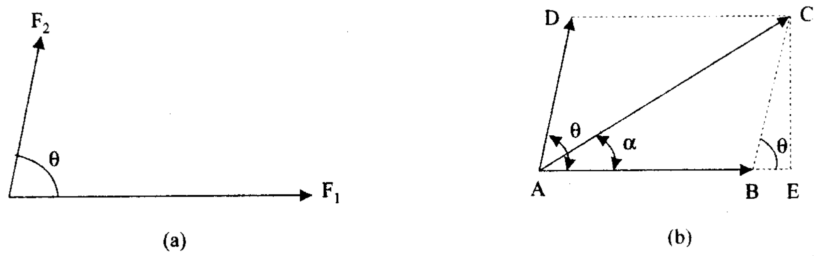


Fig. 5.3

Now the resultant R of F_1 and F_2 is given by

$$R = AC$$

$$= \sqrt{AE^2 + CE^2}$$

$$= \sqrt{(AB + BE)^2 + CE^2}$$

But $AB = F_1$

$$BE = BC \cos\theta = F_2 \cos\theta$$

and $$CE = BC \sin\theta = F_2 \sin\theta$$

$$\therefore \quad R = \sqrt{(F_1 + F_2 \cos\theta)^2 + (F_2 \sin\theta)^2}$$

$$= \sqrt{F_1^2 + 2F_1F_2 \cos\theta + F_2^2 \cos^2\theta + F_2^2 \sin^2\theta}$$

$$= \sqrt{F_1^2 + 2F_1F_2 \cos\theta + F_2^2} \qquad \text{Eqn. (5.1)}$$

The inclination of the resultant to force F_1 is given by α, where

$$\tan\alpha = \frac{CE}{AE} = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}$$

Thus $$\alpha = \tan^{-1} \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta} \qquad \text{Eqn. (5.2)}$$

Particular cases:

(i) When $\theta = 90°$ [Ref. Fig. 5.4(a)] $\quad R = \sqrt{F_1^2 + F_2^2}$

(ii) When $\theta = 0°$ [Ref. Fig. 5.4(b)] $\quad R = \sqrt{F_1^2 + 2F_1F_2 + F_2^2} = F_1 + F_2$

(iii) When $\theta = 180°$ [Ref. Fig. 5.4(c)] $\quad R = \sqrt{F_1^2 - 2F_1F_2 + F_2^2} = F_1 - F_2$

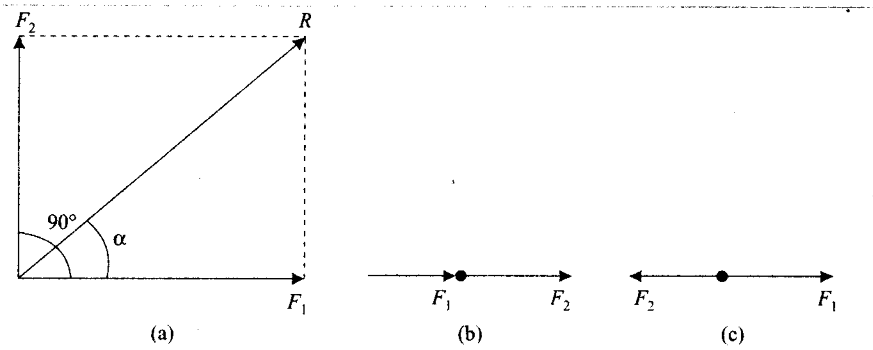
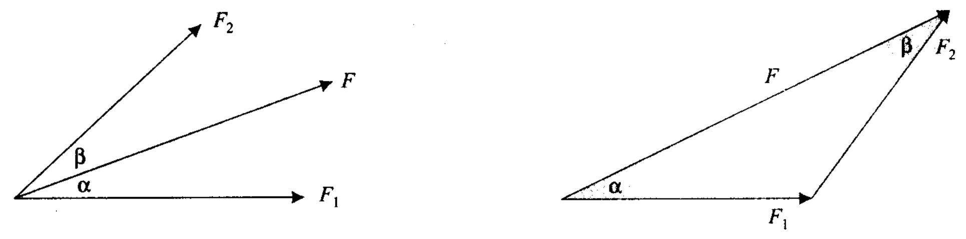


Fig. 5.4

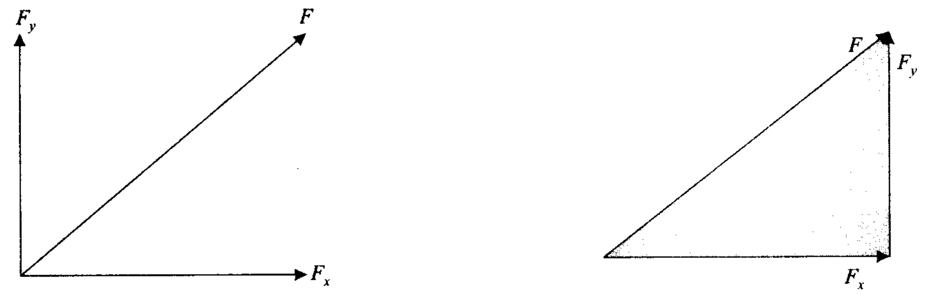
From cases (ii) and (iii) it is clear that when the forces acting on a body are collinear, their resultant is equal to the algebraic sum of the forces.

5.4 PRINCIPLE OF RESOLVED PARTS

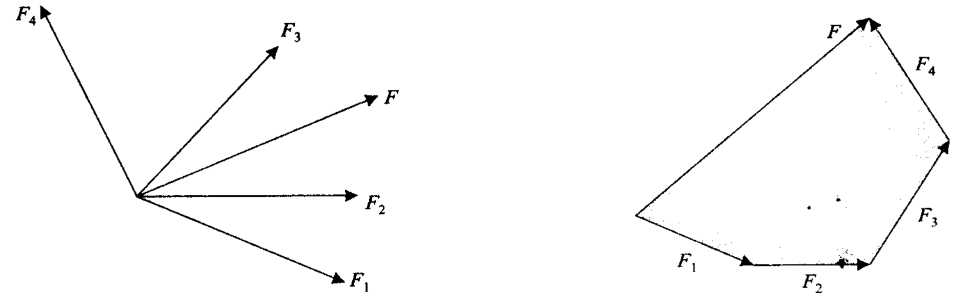
In chapter 4, we have seen resolution is exactly opposite process of composition of forces. Hence exactly opposite process of composition can be employed to get the resolved parts of a given force. For this the reverse of triangle law and polygonal law can be used.



(a)



(b)



(c)

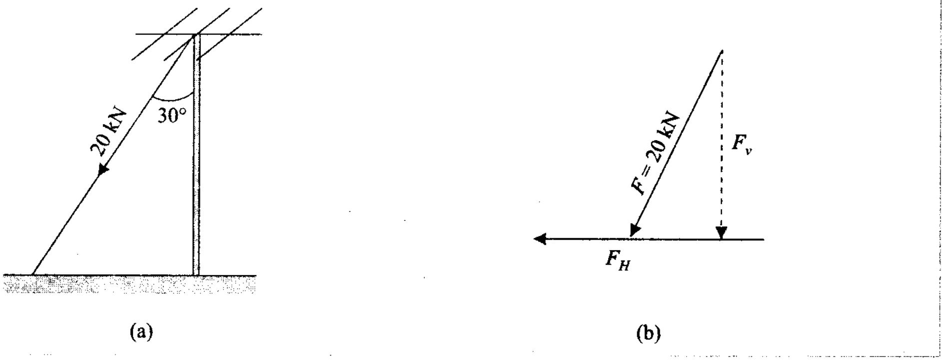
In Fig. 5.5(a) the given force F is resolved into two components making angles α an β with F.

In Fig. 5.5(b) the force F is resolved into its rectangular components F_x and F_y.

In Fig. 5.5(c) the force F is resolved into four components F_1, F_2, F_3 and F_4.

It may be noted that all component forces act on the same point as the given force. For further study resolution of given force into its two rectangular components will be more useful. From the case shown in Fig. 5.5(b), it may be noted that **to get the rectangular components of a given force travel from the tail of the force line to arrowhead in the direction of the required coordinates. Then the direction of travel gives the direction of component forces. From the triangle law the magnitude of the forces can be calculated.**

Example 5.1 The guy wire of an electric pole shown in Fig. 5.6(a) makes an angle of 30° to the pole and is subjected to a force of 20 kN. Find the vertical and the horizontal components of the force.



Solution. Figure 5.6(b) shows the resolution of the given 20 kN force into its vertical and horizontal direction. From the figure it may be observed that

$F_V = F \cos 30 = 20 \cos 30 =$ **17.34 kN (downward)** **Ans.**

and $F_H = F \sin 30 = 20 \sin 30 =$ **10.00 kN (towards left)**

Ans.

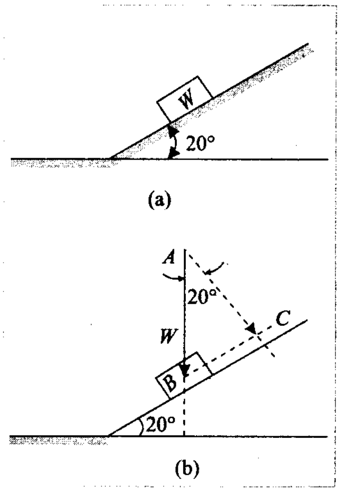
Example 5.2 A block weighing $W = 10$ kN is resting on an inclined plane as shown in Fig. 5.7(a). Determine its components normal to and parallel to the inclined plane.

Solution. The plane makes an angle of 20° to the horizontal. Hence normal to the plane makes an angle of 20° with the vertical as shown in Fig. 5.7(b). In this figure, if *AB* represents the given force to some scale, *AC* represents the normal component and *CB* represents component parallel to the plane.

Referring to the triangle of forces *ABC*, we get

Normal component $= AC = W \cos 20$
$= 10 \cos 20$
= 9.4 kN (Thrust on the plane) **Ans.**

Parallel component $= CB = W \sin 20$
$= 10 \sin 20$
= 3.42 kN (down the plane) **Ans.**



In this analytical method of a finding resultant of a number of concurrent forces, first, components of each force are found in two mutually perpendicular directions. Then the components in each directions are added algebraically to get two components. These two components which are in mutually perpendicular directions are combined to get the required resultant.

Let F_1, F_2, F_3 and F_4 shown in Fig. 5.8(a) be the system of four forces, the resultant of which is required.

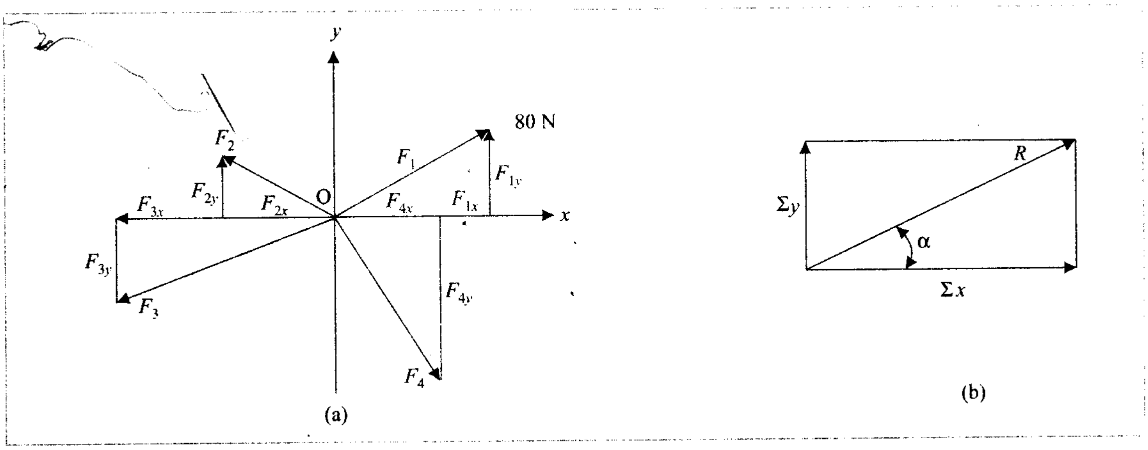


Fig. 5.8

The procedure to get the resultant is given below:

Step 1. Find the component of all forces in x and y directions. Thus F_{1x}, F_{2x}, F_{3x}, F_{4x}, F_{1y}, F_{2y}, F_{3y} and F_{4y} are obtained.

Step 2. Find the algebraic sum of the component forces in x and y directions.

$$\sum F_x = F_{1x} + F_{2x} + F_{3x} + F_{4x}$$

$$\sum F_y = F_{1y} + F_{2y} + F_{3y} + F_{4y}$$

[*Note:* In the above case F_{2x}, F_{3x}, F_{3y}, and F_{4y} are having negative values]

Step 3. Now the system of forces is equal to two mutually perpendicular forces, namely, ΣF_x and ΣF_y as shown in Fig. 5.8(b). Since these two forces are perpendicular to each other,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} \qquad \text{Eqn. (5.3)}$$

and

$$\alpha = \tan^{-1} \frac{\Sigma F_y}{\Sigma F_x} \qquad \text{Eqn. (5.4)}$$

Note:

$$R \cos \alpha = \Sigma F_x \qquad \text{Eqn. (5.5)}$$

and

$$R \sin \alpha = \Sigma F_y \qquad \text{Eqn. (5.6)}$$

i.e., ΣF_x and ΣF_y are the x and y components of the resultant.

Example 5.3 The resultant of two forces, one of which is double the other is 260 N. If the direction of the larger force is reversed and the other one remains unaltered, the resultant reduces to 180 N. Determine the magnitude of the forces and the angle between the two forces.

Solution. Let the magnitude of the smaller force be F. Hence magnitude of the larger force is $2F$.

Thus $$F_1 = F \text{ and } F_2 = 2F$$

Let θ be the angle between the two forces.

∴ From case 1, we get

$$R = \sqrt{F_1^2 + 2F_1F_2 \cos\theta + F_2^2} = 260$$

i.e., $$(F)^2 + 2(F)(2F)\cos\theta + (2F)^2 = 260^2$$

i.e., $$5F^2 + 4F^2\cos\theta = 67600 \quad \text{...(i)}$$

From case 2, we have

$$\sqrt{F_1^2 + 2F_1F_2 \cos(180 + \theta) + F_2^2} = 180^2$$

$$(F)^2 - 2(F)(2F)\cos\theta + (-2F)^2 = 180^2$$

i.e., $$5F^2 - 4F^2\cos\theta = 32400 \quad \text{...(ii)}$$

Adding equations (i) and (ii), we get

$$10F^2 = 100000$$

∴ $$F = 100 \text{ N}$$

Hence $$\mathbf{F_1 = 100 \text{ N and } F_2 = 200 \text{ N}}$$ **Ans.**

Substituting the value of F_1 and F_2 in eqn (i),

we get, $$5(100)^2 + 4(100)^2\cos\theta = 67600$$

∴ $$\cos\theta = 0.44$$

∴ $$\theta = 63.9°$$ **Ans.**

Example 5.4 Determine the resultant of the three forces acting on a hook shown in Fig. 5.9.

Solution.

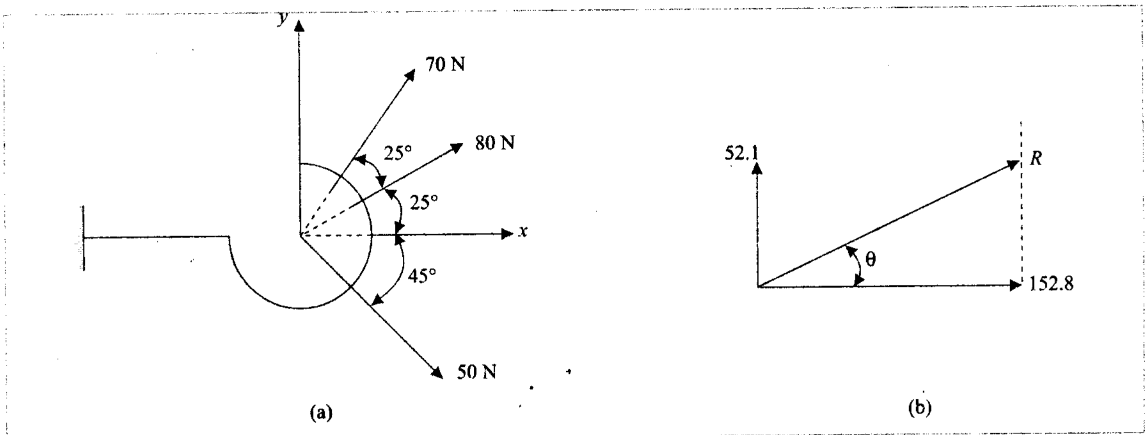


Fig. 5.9

Force	x-component	y-component
F_1 =70 N	70 cos 50 = 45.00	70 sin 50 = 53.6 N
F_2 = 80 N	80 cos 25 = 72.5 N	80 sin 25 = 33.8 N
F_3 = 50 N	50 cos 45 = 35.4 N	- 50 sin 45 = -35.4 N

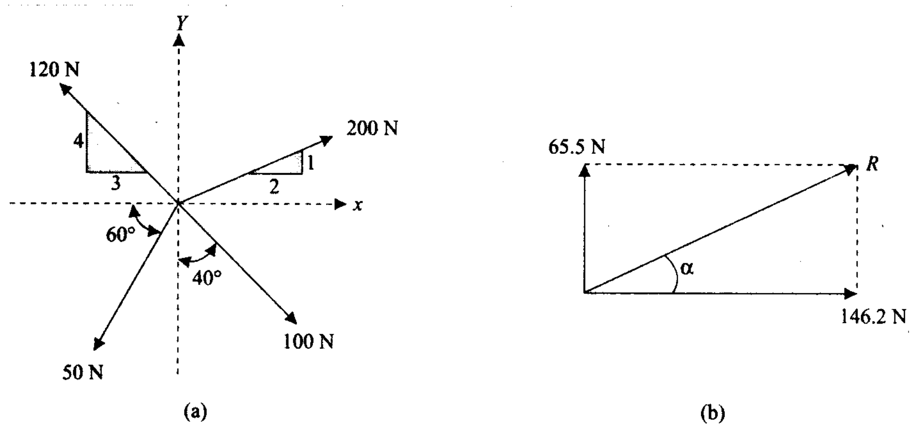
$$\sum F_x = 45.0 + 72.5 + 35.4 = 152.8 \text{ N}$$

$$\sum F_y = 53.6 + 33.8 - 35.4 = 52.1 \text{ N}$$

$\therefore$ $$R = \sqrt{152.8^2 + 52.1^2} = \mathbf{161.5\ N}$$ **Ans.**

$$\alpha = \tan^{-1}\frac{52.1}{158.8} = \mathbf{18.83°}$$ **Ans.**

Example 5.5 A system of four forces acting at a point on a body is as shown in Fig. 5.10(a). Determine the resultant.



Solution. If θ_1 is the inclination of 200 N force to x-axis, then

$$\tan\theta_1 = \frac{1}{2} \quad \therefore \theta_1 = 26.565°$$

Similarly inclination of 120 N force to x-axis is given by

$$\tan\theta_2 = \frac{4}{3} \quad \text{i.e., } \theta_2 = 53.13°.$$

$$\sum F_x = 200 \cos 26.565 - 120 \cos 53.13 - 50 \cos 60 + 100 \sin 40 = 146.2 \text{ N}$$

$$\sum F_y = 200 \sin 26.565 + 120 \sin 53.13 - 50 \sin 60 - 100 \cos 40 = 65.5 \text{ N}$$

$\therefore$ $R = \sqrt{146.2^2 + 65.5^2} = 160.2$ N **Ans.**

$\alpha = \tan^{-1} \dfrac{65.5}{146.2}$ = **24.1° as shown in Fig. 5.10(b).** **Ans.**

Example 5.6 A system of forces acting on a body resting on an inclined plane is as shown in Fig. 5.11. Determine the resultant force if $\theta = 60°$, $W = 1000$ N, vertically downward, $N = 500$ Newton acting normal to the plane, $F = 100$ N, acting down the plane and $T = 1200$ N, acting parallel to the plane.

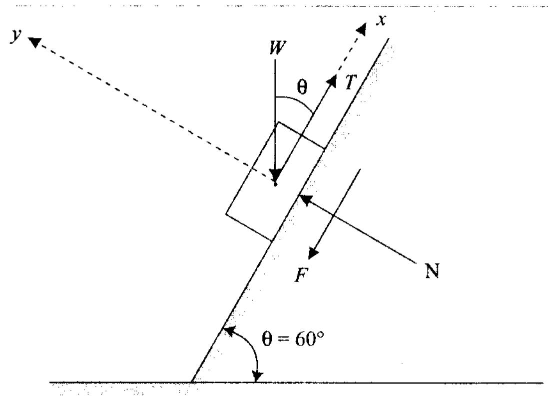


Fig. 5.11

Solution. In this problem coordinates are taken parallel to and perpendicular to inclined plane, since they are more convenient. Noting that W makes an angle θ with y-axis.

$$\sum F_x = T - F - W \sin \theta$$

$$= 1200 - 100 - 1000 \sin 60$$

$$= 234.0 \text{ N}$$

$$\sum F_y = N - W \cos 60 = 500 - 1000 \cos 60 = 0$$

$\therefore$ **Resultant is a force of magnitude 234 N directed up the plane.** **Ans.**

Example 5.7 Two forces acting on a body are 500 N and 1000 N as shown in Fig. 5.12(a). Determine the third force F such that the resultant of all the three forces is 1000 N, directed at 40° to x-axis.

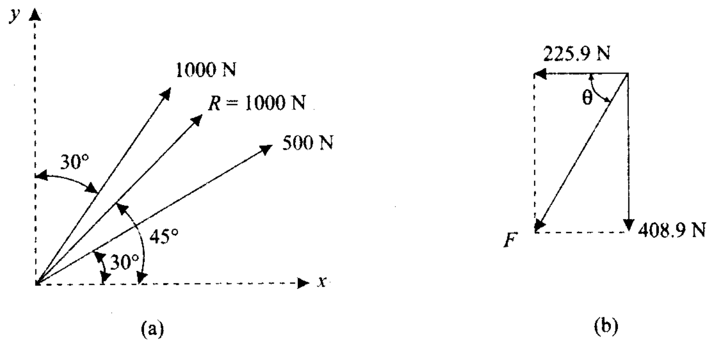


Fig. 5.12

Solution. Let the third force F make an angle θ with the x-axis. Then,

$$R \cos \alpha = \sum F_x, \quad \text{gives}$$

$$1000 \cos 45 = 500 \cos 30 + 1000 \sin 30 + F \cos \theta$$

$\therefore$ $$F \cos \theta = -225.9 \text{ N}$$

Similarly,

$$R \sin 45 = \sum F_y \quad \text{gives,}$$

$$1000 \sin 45 = 500 \sin 30 + 1000 \cos 30 + F \sin \theta$$

$\therefore$ $$F \sin \theta = -408.9 \text{ N}$$

$\therefore$ $$F = \sqrt{225.9^2 + 408.9^2} = \mathbf{467.2\ N}$$ **Ans.**

and $$\tan \theta = \frac{-408.9}{-225.9} = 1.810$$

$\therefore$ θ = **61.08° as shown in Fig. 5.12(a)** **Ans.**

Example 5.8 Three forces acting at the centre of gravity of a block are shown in Fig. 5.13. The direction of 300 N forces may vary but the angle between them is always 40°. Determine the value of θ for which the resultant of the three forces is directed parallel to the plane.

Solution. Let x and y axes be selected as shown in Fig. 5.13. If the resultant is directed along x-axis, the component of reaction in y-direction should be zero.

i.e., $$\sum F_y = 0$$

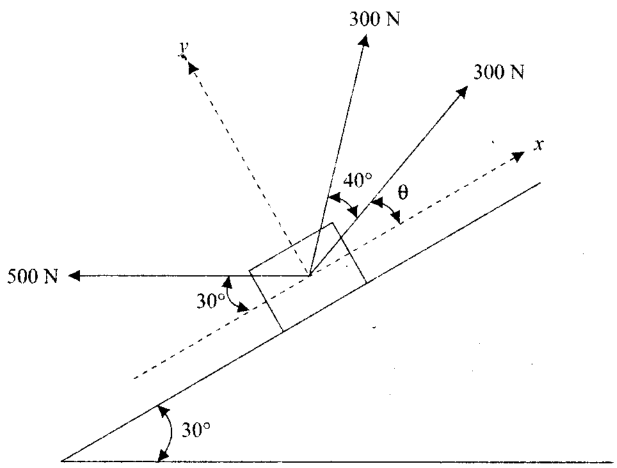


Fig. 5.13

i.e., $300 \sin \theta + 300 \sin (40 + \theta) - 500 \sin 30 = 0$

$$\sin \theta + \sin (40 + \theta) = 0.833$$

i.e., $2 \sin \frac{40 + \theta + \theta}{2} \cos \frac{40 + \theta - \theta}{2} = 0.833$

i.e., $2 \sin (20 + \theta) \cos 20 = 0.833$

$$\sin (20 + \theta) = 0.443$$

i.e., $20 + \theta = 26.31$

$\therefore$ $\theta = \mathbf{6.31°}$ **Ans.**

Note: $\sin A \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$

Important Definitions and Formulae

1. The single force which will have the same effect as the system of forces is defined as resultant.
2. Parallelogram law of forces states that if two forces acting on a body at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram, their resultant is represented in magnitude and direction by the diagonal of the parallelogram which passes through the point of intersection of the two sides representing the forces.
3. Triangle law of forces may be stated as 'if two forces acting on a body are represented one after the other by the sides of a triangle, their resultant is represented by the closing sides of the triangle taken from the first point to the last point.

4. Polygon law of forces may be stated as 'if a number of concurrent forces acting simultaneously on a body are represented in magnitude and direction by the sides of a polygon, taken in order, then the resultant is represented in magnitude and direction by the closing line of the polygon, taken from the first point to the last point.
5. If a force F makes angle θ with x-axis

$$F_x = F\cos\theta$$
$$F_y = F\sin\theta$$

6. $$\sum F_x = R\cos\alpha$$

$$\sum F_y = R\sin\alpha,$$ where α is angle made by resultant with x-axis

$$R = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2}$$

$$\alpha = \tan^{-1}\frac{\sum F_y}{\sum F_x}$$

Problems for Exercise

1. The resultant of two forces one of which is 3 times the other is 300 N. When the direction of the smaller force is reversed the resultant is 200 N. Determine the two forces and the angle between them.

[**Ans.** F_1 = 80.6 N, F_2 = 241.8 N, θ = 50.13°]

2. A body is subjected to the three forces as shown in Fig. 5.14. If possible determine the direction of the force F so that the resultant is in x-direction when (a) F = 5000 N, (b) F = 3000 N.

[**Ans.** (a) 36.87°, (b) not possible]

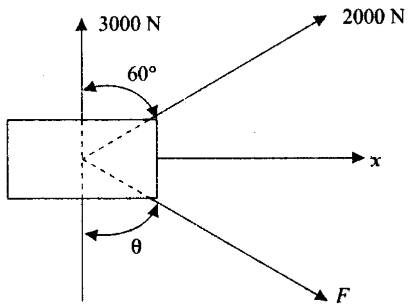
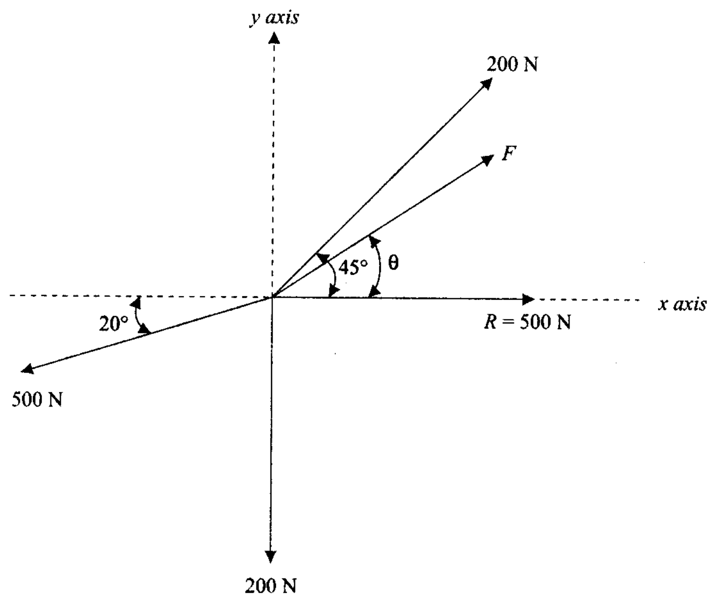


Fig. 5.14

3. The four coplanar forces acting at a point are as shown in Fig. 5.15. One of the forces is unknown and its magnitude is as shown by F. The resultant is 500 N and is along x-axis. Determine the force F and its inclination θ with x-axis.

[**Ans.** F = 859.64 N θ = 15.49°]



CHAPTER 6

Composition of Coplanar Non-concurrent Force System

As explained in Chapter 4, if all the forces in a system lie in a single plane and the lines of action of all the forces do not pass through a single point, the system is said to be coplanar non-concurrent force system. Varignon's principle of moments is very useful in determining the resultant of such force system. Hence first Varignon's principle of moments is explained and then a number of numerical problems are solved.

6.1 VARIGNON'S PRINCIPLE OF MOMENTS

French mathematician Varignon (1654-1722) gave the following theorem which is also known as principle of moments.

The algebraic sum of moments of a system of coplanar forces about a moment centre is equal to the moment of their resultant force about the same moment centre.

Proof: Referring to Fig. 6.1, let R be the resultant of forces F_1 and F_2 and 'O' be the moment centre. Let d, d_1 and d_2 be the moment arms of the forces R, F_1 and F_2 respectively. Then in this case we have to prove

$$Rd = F_1 d_1 + F_2 d_2 \qquad \text{Eqn. (6.1)}$$

Join OA and consider it as y-axis. Draw x-axis to it with A as origin [Ref. Fig. 6.1(b)]. Let resultant make an angle θ with x-axis. Noting that angle AOB is also θ, we can write

$$Rd = R \times AO \cos \theta$$

$$= AO \times (R \cos \theta)$$

$$= AO \times R_x \qquad \text{...(i)}$$

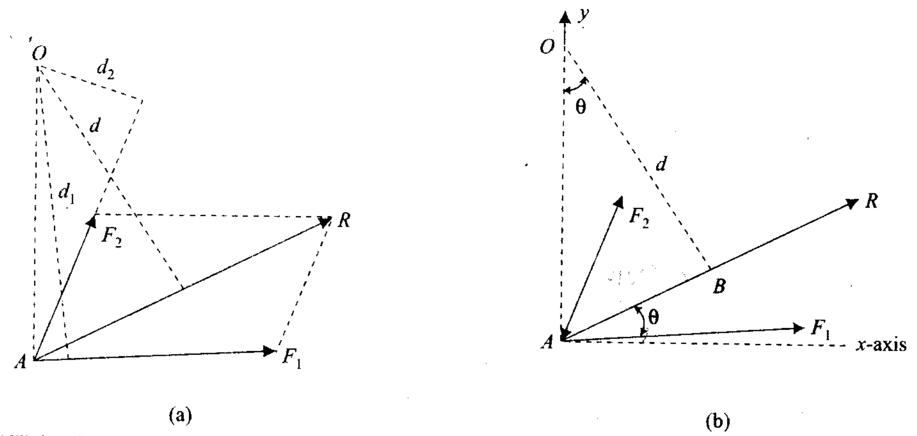


Fig. 6.1

where R_x denotes the component of R in x-direction. Similarly, if F_{1x} and F_{2x} are the components of F_1 and F_2 in x-direction,
then,

$$F_1 d_1 = AO \times F_{1x} \quad \text{...(ii)}$$

and

$$F_2 d_2 = AO \times F_{2x} \quad \text{...(iii)}$$

From eqns. (ii) and (iii), we get

$$F_1 d_1 + F_2 d_2 = AO \times (F_{1x} + F_{2x})$$
$$= AO \times R_x \quad \text{...(iv)}$$

From eqns. (i) and (iv), we observe

$$F_1 d_1 + F_2 d_2 = Rd$$

Thus we find sum of the moment of forces about a moment centre is same as moment of their resultant about the same centre.

If a system of forces consists of more than two forces, the result can be extended as given below:

Let F_1, F_2, F_3 and F_4 be four concurrent forces and R be their resultant. Referring to Fig. 6.2, d_1, d_2, d_3, d_4 and 'a' be moment arms of F_1, F_2, F_3, F_4 and R about moment centre 'O'.

If R_1 is the resultant of forces F_1 and F_2 and its moment arm is a_1, then from the above proof for two force system, we get

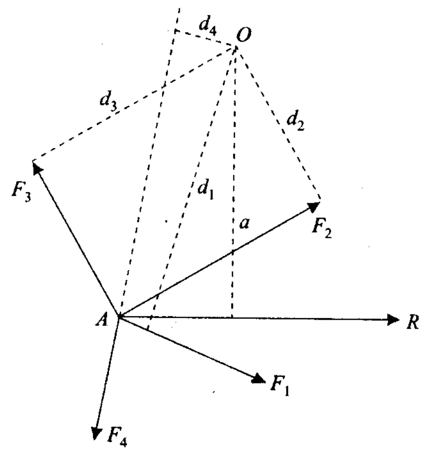


Fig. 6.2

$$R_1 a_1 = F_1 d_1 + F_2 d_2$$

If R_2 is the resultant of R_1 and F_3 and moment arm from 'O' is a_2, we can say

$$R_2a_2 = R_1a_1 + F_3d_3$$
$$= F_1d_1 + F_2d_2 + F_3d_3$$

Similarly, considering R_2 and F_4, we can write

$$Ra = R_2a_2 + F_4d_4$$
$$= F_1d_1 + F_2d_2 + F_3d_3 + F_4d_4 \qquad \text{Eqn. (6.2)}$$

Thus, the moment of the resultant of a number of forces about a moment centre is equal to the sum of the moments of its component forces about the same moment centre.

Example 6.1 Determine the moment of 100 N force acting at B about moment centre A [Ref. Fig. 6.3].

Solution. 100 N force may be resolved into its horizontal component and vertical components of magnitude 100 cos 60° and 100 sin 60° respectively. From Varignon's theorem, moment of 100 N force about moment centre A is equal to sum of moments as its components about A. Taking clockwise moment as positive,

$$M_A = 100 \cos 60 \times 300 - 100 \sin 60 \times 500$$
$$= -28301 \text{ N-mm.}$$

i.e., **M_A = 28301 N-mm, anticlockwise** **Ans.**

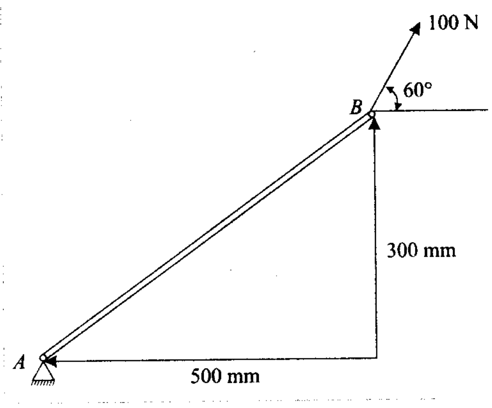


Fig. 6.3

Example 6.2 What will be y-intercept of 500 N force shown in Fig. 6.4, if its moment about A is 800 N-mm?

Solution. 500 N force is shifted to point B along the line of action (law of transmissibility) and then it is resolved into its x and y components F_x and F_y as shown in figure. Noting that

$$\cos\theta = \frac{4}{5} \text{ and } \sin\theta = \frac{3}{5},$$

$$F_x = 5000 \cos\theta = 5000 \times \frac{4}{5} = 4000 \text{ N}$$

and $$F_y = 5000 \sin\theta = 5000 \times \frac{3}{5} = 3000 \text{ N}$$

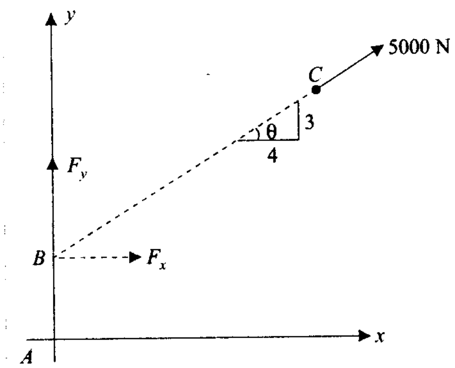


Fig. 6.4

By Varignon's theorem, moment of 5000 N force about A is equal to the moment of its component forces about the same point.

$$8000 = 4000 \times y + 3000 \times 0$$

$\therefore$ $$y = 2 \text{ m.}$$ **Ans.**

6.2 RESULTANT OF NON-CONCURRENT FORCE SYSTEM

Resultant of a force system is the one which will have the same rotational and translatory effect as the given system of forces is having. It may be a single force, a pure moment or a force and a moment.

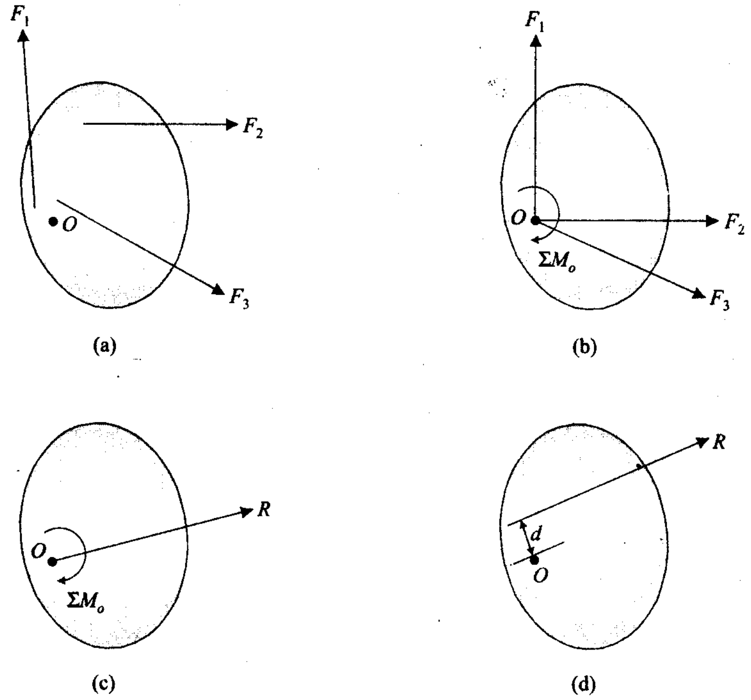


Fig. 6.5

Let F_1, F_2 and F_3 shown in Fig. 6.5(a) constitute a system of forces acting on a body. Each force can be replaced by a force of same magnitude, acting in the same direction at point 'O' and a moment about 'O'. Thus the given system in Fig. 6.5(a) is equal to the system shown in Fig. 6.5(b), where ΣM_o is algebraic sum of the moments of the given forces about 'O'. At 'O', the concurrent forces F_1, F_2 and F_3 can be combined to get the resultant force R. Now the resultant of the given system is equal to a force R at 'O' and a moment ΣM_o as shown in Fig. 6.5(c).

The force R and ΣM_o shown in Fig. 6.5(c) can be replaced by a single force R acting at a distance 'd' from 'O' such that the moment produced by this force about 'O' is equal to ΣM_o [Ref. Fig. 6.5(d)].

Thus we get a single force R acting at a distance 'd' from the point 'O' which gives the same effect as the constituent forces of the system. Mathematically

$$\left.\begin{aligned} R &= \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2} \\ \alpha &= \tan^{-1}\frac{\sum F_y}{\sum F_x}, \qquad \text{and} \\ d &= \frac{\sum M_o}{R} \end{aligned}\right\} \quad \text{Eqn. (6.3)}$$

where ΣF_x – algebraic sum of components of all forces in x-direction,
ΣF_y – algebraic sum of components of all forces in y-direction,
α – inclination of the resultant R to x-axis,
ΣM_o – algebraic sum of all the forces about a point 'O'.
and d – is the distance of the line of action of resultant from point O.

Note:

(i) R is maked at distance 'd' from point 'O' such that it produces the same direction of the moment as ΣM_o.

(ii) Sometimes the value of ΣF_x and ΣF_y may become zero, but ΣM_o may exist. This means that the resultant of the system gets reduced to a pure moment.

6.3 *x* AND *y* INTERCEPTS OF RESULTANT

In the previous article, we have seen how to find a point of application on the resultant by finding the perpendicular distance 'd' from the reference point 'O'. Many times it is convenient to locate the point of intersection of the line of action of the resultant on x or y axis through the reference point 'O'.

Let d be the distance of the resultant from 'O' and α be its inclination to x-axis [Ref. Fig. 6.6]. Then x and y intercepts of the resultant are given by

$$x = \frac{d}{\sin\alpha} \quad \text{and} \quad y = \frac{d}{\cos\alpha} \qquad \text{Eqn. 6.4}$$

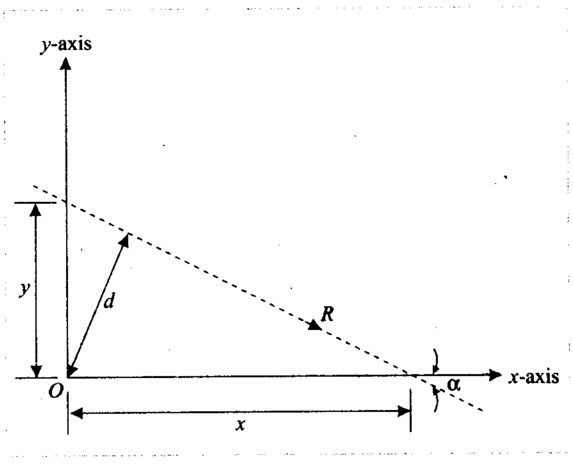


Fig. 6.6

Another method of finding the intercepts is given below:

Let $R_x = \Sigma F_x$ and $R_y = \Sigma F_y$ be the components of the resultant R in x and y directions. Considering the moment of R about O as the sum of its moments of its components A about O (Varignon's theorem), we get (Ref. Fig. 6.7),

$$Rd = \sum M_o$$

$$R_x \times O + R_y x = \sum M_o$$

$$\therefore \quad x = \frac{\sum M_o}{R_y} = \frac{\sum M_o}{\sum F_y} \qquad \text{Eqn. 6.5}$$

Similarly, resolving the resultant into its components at B (Ref. Fig. 6.7) and then applying Varignon's theorem for moments about 'O' it can be shown that

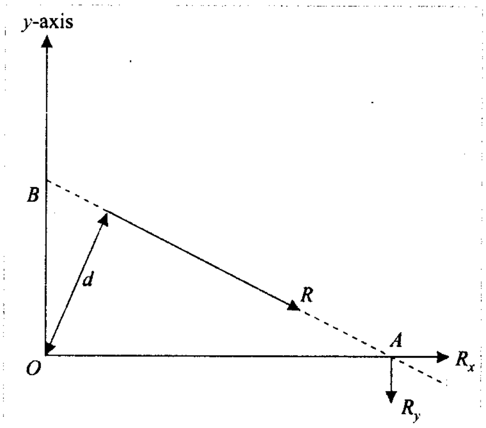


Fig. 6.7

$$y = \frac{\sum M_o}{R_x} = \frac{\sum M_o}{\sum F_x} \qquad \text{Eqn. (6.6)}$$

Note: *A resultant in non-concurrent force system is completely determined when its magnitude (R), direction (α) and a point on its line of action (d, x or y intercept) are determined.*

Example 6.3 Find the resultant of the force system shown in Fig. 6.8(a) acting on a lamina of equilateral triangular shape.

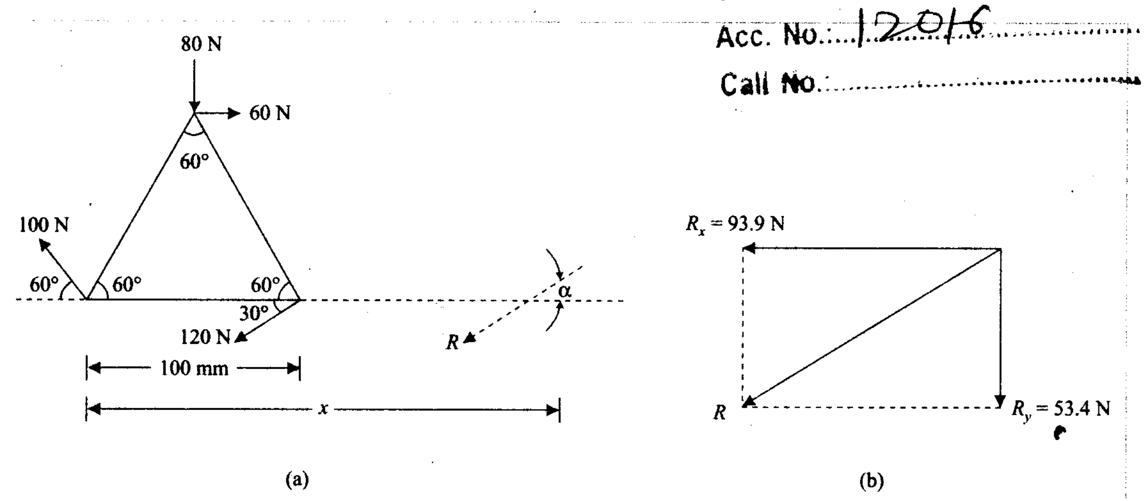


Fig 6.8

Solution.

$$R_x = \sum F_x = 60 - 100 \cos 60 - 120 \cos 30$$

$$= -93.9 \text{ N} = \overleftarrow{93.9} \text{ N}$$

$$R_y = \sum F_y = -80 + 100 \sin 60 - 120 \sin 30$$

$$= -53.40 = 53.40 \text{ N} \downarrow$$

$\therefore$ $$R = \sqrt{93.92^2 + 53.40^2}$$

i.e., $\mathbf{R = 108.0\ N}$ **Ans.**

$$\alpha = \tan^{-1} \frac{R_y}{R_x} = \tan^{-1} \frac{53.40}{93.9} = -2$$

i.e., $\alpha = 29.60°$, as shown in Fig. 6.8(b) **Ans.**

Let x be the intercept on x-axis from A. Then taking moment about A,

$$x = \frac{\sum M_A}{R_y} = \frac{80 \times 100 \cos 60 + 60 \times 100 \sin 60 + 120 \sin 30 \times 100}{53.40}$$

i.e., x = 284.6 mm, as shown in Fig. 6.8(a). **Ans.**

Example 6.4 Find the resultant of the system of coplanar forces acting on a lamina as shown in Fig. 6.9(a). Each square has a side of 10 mm.

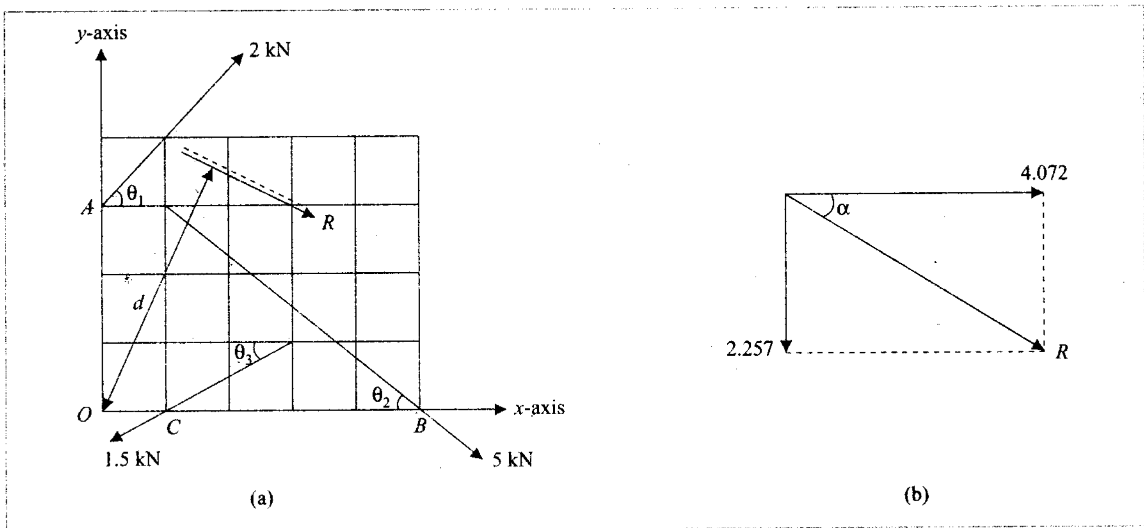


Fig. 6.9

Solution. If θ_1, θ_2 and θ_3 are the inclinations of forces 2 kN, 5 kN and 1.5 kN with respect to x-axis, then

$$\tan \theta_1 = \frac{10}{10} = 1 \qquad \therefore \theta_1 = 45°$$

$$\tan\theta_2 = \frac{30}{40} \qquad \therefore\ \theta_2 = 36.87°$$

$$\tan\theta_3 = \frac{10}{20} \qquad \therefore\ \theta_3 = 26.565°$$

$$R_x = \sum F_x = 2\cos 45 + 5\cos 36.87 - 1.5\cos 26.565$$

$$= 4.072\text{ kN}$$

$$R_y = \sum F_y = 2\sin 45 - 5\sin 36.87 - 1.5\sin 26.565$$

$$= -2.257\text{ kN}$$

$\therefore$ $$R = \sqrt{4.072^2 + (-2.257)^2} = \mathbf{4.655\ kN}$$ **Ans.**

$$\alpha = \tan^{-1}\frac{2.257}{4.072} = \mathbf{29°}$$, **as shown in Fig. 6.9(b)** **Ans.**

Resolving the forces into their x and y components at A, B and C as shown in Fig. 6.9(a) and then finding their moment about 'O', we get,

$$\sum M_o = 2\cos 45 \times 30 + 5\sin 36.87 \times 50 + 1.5\sin 26.565 \times 10$$

$$= 199.13\text{ kN-mm}$$

$\therefore$ Distance d of the resultant from O is given by

$$d = \frac{199.13}{R} = \frac{199.13}{4.655} = \mathbf{42.8\ mm}$$ **as shown in Fig. 6.9(b).** **Ans.**

Example 6.5 The system of forces acting on a bell crank is shown in Fig. 6.10(a). Determine the magnitude, direction and the point of application of the resultant.

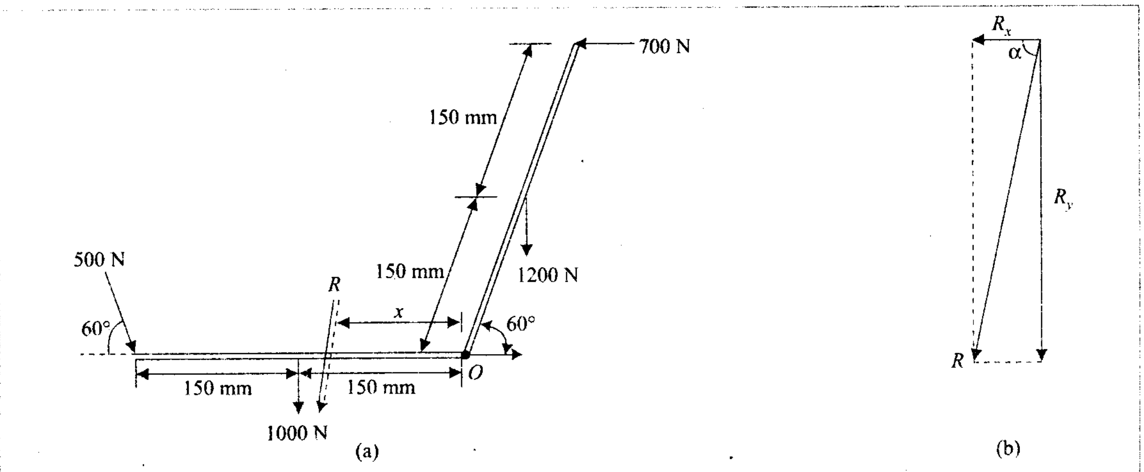


Fig. 6.10

Solution.

$$R_x = \sum F_x = 500 \cos 60 - 700 = -450 \text{ N} = \overleftarrow{450} \text{ N}$$

$$R_y = \sum F_y = -500 \sin 60 - 1000 - 1200 = -2633 \text{ N} = 2633 \text{ N}\downarrow$$

$\therefore$ $$R = \sqrt{450^2 + 2633^2} = \mathbf{2671.2\ N}$$ **Ans.**

$$\alpha = \tan^{-1} \frac{2633}{450} = \mathbf{80.30°}\text{, **as shown in Fig 6.10(b)**}$$ **Ans.**

Let the point of application of the resultant be at a distance x from 'O' along the horizontal arm. Then

$$x = \frac{\sum M_o}{R_y} = \frac{-500 \sin 60 \times 300 - 1000 \times 150 + 1200 \times 150 \cos 60 - 700 \times 300 \sin 60}{2633}$$

$\therefore$ **x = 141.2 mm as shown in Fig. 6.10(a)** **Ans.**

Example 6.6 Various forces to be considered for the stability analysis of a dam are shown in Fig. 6.11. The dam is safe, if the resultant of forces pass through middle third of the base. Verify whether the dam is safe.

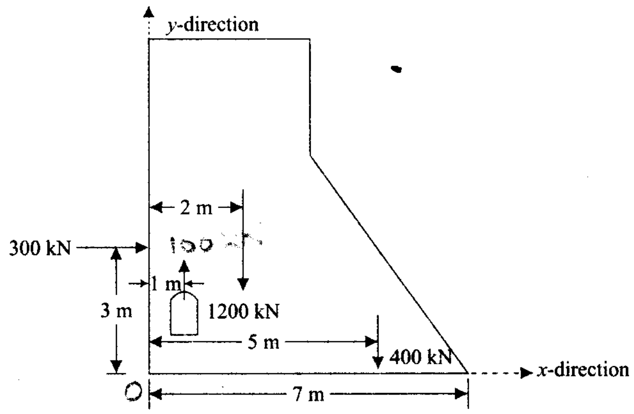


Fig. 6.11

Solution.

$$R_x = \sum F_x = 300 \text{ kN}$$

$$R_y = \sum F_y = 100 - 1200 - 400 = -1500 \text{ kN} = 1500 \text{ kN}\downarrow$$

Let the resultant pass through the base at a distance x from 'O'. Then

$$x = \frac{\sum M_o}{R_y} = \frac{300 \times 3 - 1000 \times 1 + 1200 \times 2 + 400 \times 5}{1500}$$

$$= 3.467 \text{ m.}$$

The resultant lies in the middle third of the base $\left(\text{i.e., } x \text{ is between } \frac{7}{3} \text{ and } \frac{2 \times 7}{3}\right)$. Hence the **dam is safe.** **Ans.**

Example 6.7 A bracket is subjected to three forces and a couple as shown in Fig. 6.12(a). Determine magnitude, direction and the line of action of the resultant.

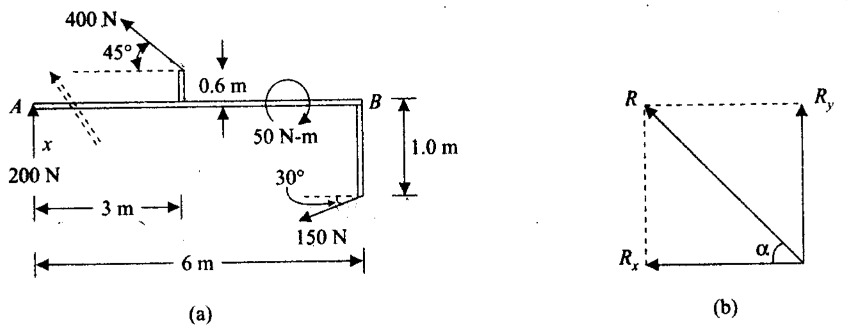


Fig. 6.12

Solution.

$$R_x = \sum F_x = -400 \cos 45 - 150 \cos 30$$

$$= -412.7 \text{ N} = 412.7 \overleftarrow{\text{N}}$$

$$R_y = \sum F_y = 200 + 400 \sin 45 - 150 \sin 30$$

$$= 407.8 \text{ N}$$

$\therefore$ $$R = \sqrt{(412.7)^2 + (407.8)^2} = \mathbf{580.2\ N}$$ **Ans.**

$$\alpha = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\frac{407.8}{412.7} = \mathbf{44.66°,\ as\ shown\ in\ Fig.\ 6.12(b)}$$ **Ans.**

Let the resultant intersect arm AB at a distance x from A.

Now,

$$\sum M_A = -400 \sin 45 \times 3 - 400 \cos 45 \times 0.6 + 50 + 150 \sin 30 \times 6 + 150 \cos 30 \times 1$$

$$= -438.3 \text{ N-m}$$

$$= 438.3 \text{ N-m, anticlockwise}$$

$\therefore$ $x = \dfrac{M_A}{R_y} = \dfrac{438.3}{407.8}$ = **1.074 m, as shown in Fig. 6.12(a)** **Ans.**

Important Definitions and Formulae

Varignon's principle of moments may be stated as in a concurrent force system moment of the resultant about a moment centre is equal to the sum of the moment of the component forces about the same point. It also means moment of a force about a moment centre is the same as sum of moment of component forces about the same moment centre.

Formulae:

1. Resultant of non-concurrent force system is given by

$$R = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2}$$

$$\alpha = \tan^{-1} \frac{\sum F_y}{\sum F_x}$$

$$d = \frac{\sum M_o}{R}$$

where d is the perpendicular distance of line of action of resultant from the moment centre 'O'.

2. Instead of locating the line of action of the resultant from point 'O' by its perpendicular distance d, it may be located by its x or y intercepts also.

$$x = \frac{\sum M_o}{R_y}$$

and

$$y = \frac{\sum M_o}{R_x}$$

Problems for Exercise

1. Determine the magnitude, direction and position of the resultant force with reference to point A for the non-coplanar force system shown below in Fig. 6.13.

[**Ans.** $\Sigma F_x = 100$ N, $\Sigma F_y = 100$ N, $R = 100\sqrt{2}$ N, $\alpha = 45°$, y intercept = 4 m, i.e., reaction passes through B]

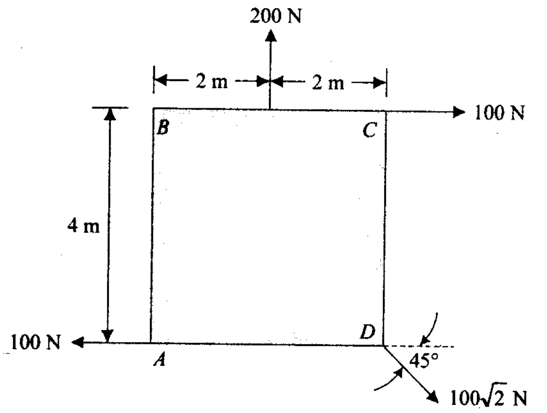


Fig. 6.13

2. Determine the resultant of the parallel coplanar force system shown in Fig. 6.14.

[**Ans.** $R = 800$ N, $d = 627.5$ mm]

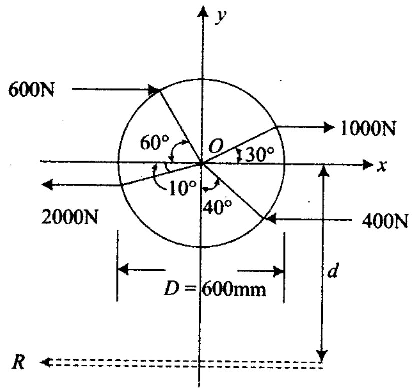


Fig. 6.14

3. Determine the magnitude, direction and the line of action of the resultant of forces shown in Fig. 6.15 which act on the plane body ABCDEFGH.

[**Ans.** $R = 23.65$ kN, $\alpha = 24.37°$, x from $A = 1.04$ m]

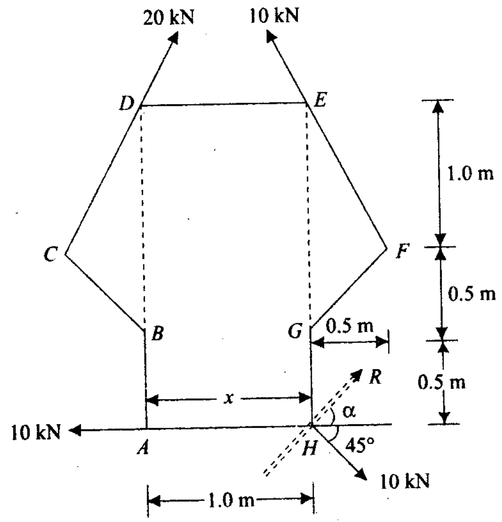


Fig. 6.15

4. Determine the resultant of the four forces acting on a body as shown in Fig. 6.16.
[**Ans.** R = 200 N, α = 60° at a distance y = 8.77 m below 'O']

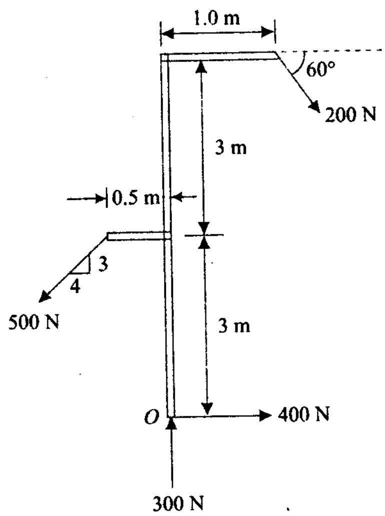


Fig. 6.16

5. An equilateral triangular plate of sides 200 mm is acted upon by four forces as shown in Fig. 6.17. Determine the magnitude and direction of the resultant and its position.
[**Ans.** R = 57.35 N, α = 6.70°, d = 11.57 mm from A]

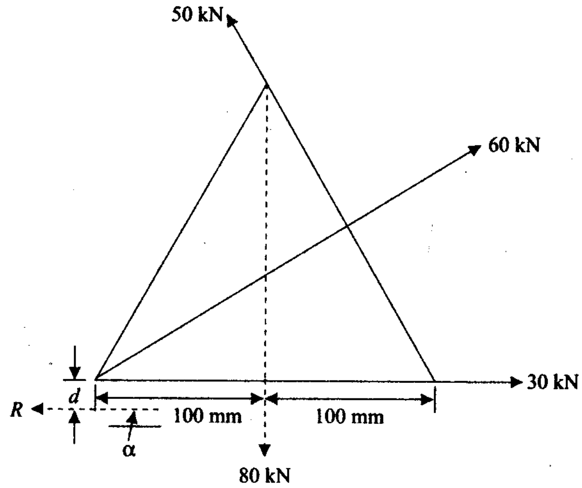


Fig. 6.17

6. A bracket is subjected to the system of forces and couples as shown in Fig. 6.18. Find the resultant of the system and the point of intersection of its line of action with (i) line *AB* (ii) line *BC* and (iii) line *CD*.

[**Ans.** R = 485.4 N, α = 34.50°, y_{BA} = 112.5 mm, x_{BC} = 163.6 mm, y_{CD} = 93.7 mm]

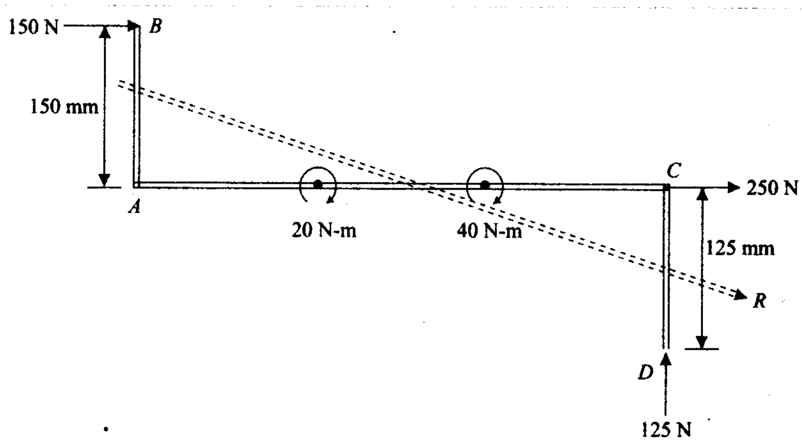


Fig. 6.18

7. Determine the resultant of the three forces acting on the dam section shown in Fig. 6.19 and locate its intersection with the base *AB*. For a safe design this intersection should occur within the middle third. Is it a safe design?

[**Ans.** Resultant intersects *AB* at 3.33 m from *A*. It is a safe design]

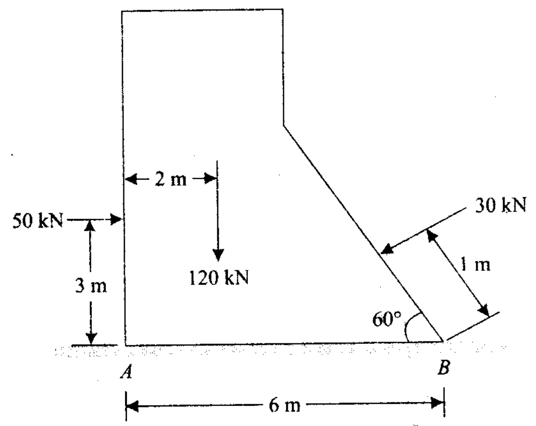


Fig. 6.19

CHAPTER 7

Centroid of Plane Figures

Centroid is an important property of the cross-section of a member which is required frequently in the analysis of many engineering problems. In this chapter the meaning of centre of gravity and centroid of plane figures is explained. Centroid of triangle, semicircle, quadrant of a circle and sector of a circle using method of integration is presented. Method of locating centroid of simple built up sections is illustrated by solving numerical problems.

7.1 CENTRE OF GRAVITY

Consider a suspended body as shown in Fig. 7.1. The weights of various parts of this body are acting vertically downward. The only upward force is the force in the string. To satisfy the equilibrium condition the resultant weight of the body W must act along the line of the string (1)-(1). Now, if the position is changed and the body is suspended again, it will reach equilibrium in a particular position. Let the line of action of resultant weight be (2)-(2) intersecting line (1)-(1) at G. It is found that if the body is suspended in any other position, the line of action of resultant weight W passes through G. This point is called the centre of gravity. Thus, *centre of gravity can be defined as the point through which resultant of force of gravity (weight) of the body acts.*

7.2 CENTRE OF GRAVITY OF FLAT PLATE

Consider a flat plate of thickness t as shown in Fig. 7.2. Let W_i be the weight of any elemental portion acting at a point (x_i, y_i). Let W be the total weight of the plate acting at the point $(\bar{x}, \bar{y})$. According to definition of centre of gravity, the point $(\bar{x}, \bar{y})$ is the centre of gravity. Now,

$$\text{Total weight } W = \sum W_i \qquad \text{Eqn. (7.1)}$$

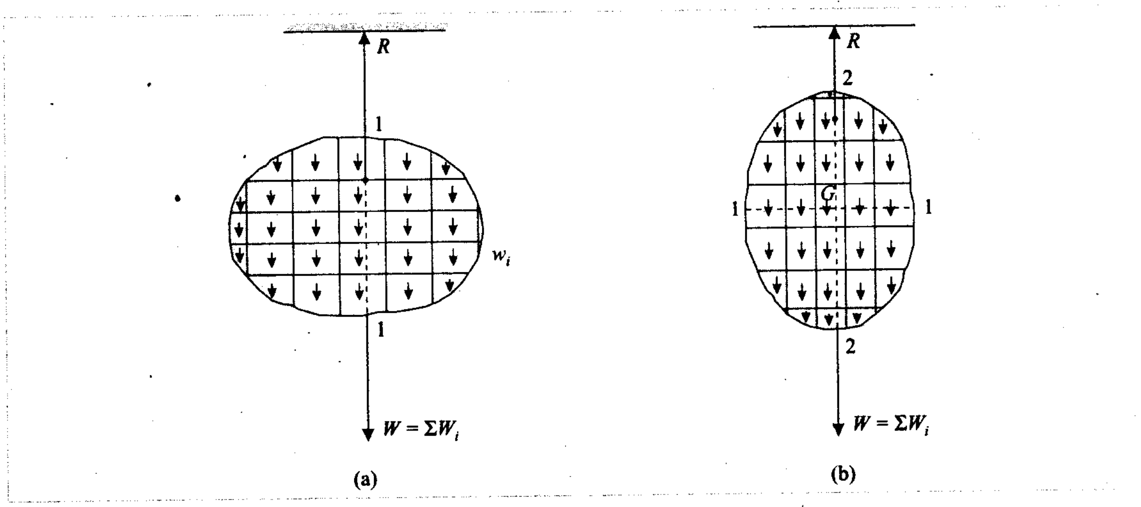


Fig 7.1

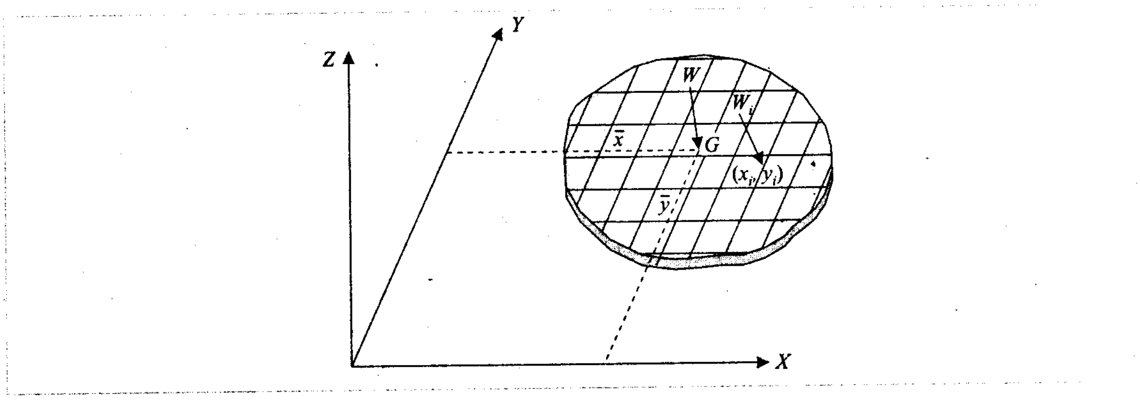


Fig 7.2

Taking moment about x-axis and equating moment of resultant to moment of component forces, we get

$$W\bar{y} = W_1y_1 + W_2y_2 + W_3y_3 + \ldots$$
$$= \Sigma W_i y_i$$

$$\therefore \qquad \bar{y} = \frac{\Sigma W_i y_i}{W} \qquad \text{Eqn. (7.2)}$$

Similary, taking moment about y-axis we get,

$$W\bar{x} = W_1x_1 + W_2x_2 + W_3x_3 + \ldots$$
$$= \Sigma W_i x_i$$

$$\bar{x} = \frac{\Sigma W_i x_i}{W} \qquad \text{Eqn. (7.3)}$$